1 a) $$\frac{1}{2}\int_{-1}^{1} f(t)\, e^{-ik\pi t}\, dt$$

$$= \frac{1}{2}\int_{-1}^{0} e^{-t} e^{-ik\pi t}\, dt + \frac{1}{2}\int_{0}^{1} e^{t} e^{-ik\pi t}\, dt$$

$$= \frac{1}{2}\left[\frac{-1}{1+ik\pi} e^{-(1+ik\pi)t}\right]_{-1}^{0} + \frac{1}{2}\left[\frac{1}{1-ik\pi} e^{(1-ik\pi)t}\right]_{0}^{1}$$

$$= \frac{1}{2}\frac{-1}{1+ik\pi}\left(1-e^{1+ik\pi}\right) + \frac{1}{2}\frac{1}{1-ik\pi}\left(e^{1-ik\pi}-1\right)$$

$$= \frac{1}{2}\left(e(-1)^k-1\right)\left[\frac{1}{1+ik\pi} + \frac{1}{1-ik\pi}\right]$$

$$= \frac{1}{2}\left(e(-1)^k-1\right)\left[\frac{2}{(1+ik\pi)(1-ik\pi)}\right]$$

$$= \left(e(-1)^k-1\right)\frac{1}{1+k^2\pi^2}$$

b) $$2f_k = a_k - ib_k$$

$$b_k = 0, \quad a_k = \frac{2(e(-1)^k-1)}{1+k^2\pi^2}$$

$$e-1+\sum_{k=1}^{\infty} a_k \cos(k\pi t)$$

1 c) $f(-1) = f(1)$

Functie is continu dus ja

d) $f'(t) = e^{t}\,\mathrm{sgn}(t)$ voor $t \in [-1,1]$

en 2-periodiek

e) $\frac{1}{2}\int_{-1}^{1} e^{2|t|}\,dt = \int_{0}^{1} e^{2t}\,dt = \frac{1}{2}(e^{2}-1)$

2 a) $\mathrm{trian}_2(2t-1) = \mathrm{trian}_1(t-\tfrac{1}{2})$

$\mathrm{trian}_1(t) \leftrightarrow \dfrac{4\sin^2(\omega/2)}{\omega^2}$

$\mathrm{trian}_1(t-\tfrac{1}{2}) \leftrightarrow e^{-i\omega/2}\,\dfrac{4\sin^2(\omega/2)}{\omega^2}$

$\dfrac{-1}{\omega^2}e^{-i\omega/2}\left(e^{+i\omega} - 2 + e^{-i\omega}\right)$

$\dfrac{-1}{\omega^2}\left(e^{i\omega/2} - 2e^{-i\omega/2} + e^{-3i\omega/2}\right)$

2 b) $-\frac{1}{2} + \cos^2(\frac{\pi}{2}t) = \frac{1}{4}e^{\pi i t} + \frac{1}{4}e^{-\pi i t}$

$$\hat{h}(\pi) = \frac{1}{\pi^2}\left(2e^{-i\pi/2} - e^{i\pi/2} - e^{-i3\pi/2} \right)$$

$$= \frac{1}{\pi^2}\left(2(-i) - i - i \right)$$

$$= -\frac{4}{\pi^2} i$$

$$\hat{h}(-\pi) = \frac{4}{\pi^2} i$$

$$y(t) = -\frac{1}{\pi^2} i e^{\pi i t} + \frac{1}{\pi^2} i e^{-\pi i t}$$

$$= \frac{2}{\pi^2}\sin(\pi t)$$

3) Met Laplace kan <u>niet</u> want $g(t)$ is niet causaal.

$$(f*g)(t) = \int_{-\infty}^{\infty} f(t-\tau)\, g(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-t+\tau}\, \mathbb{1}(t-\tau) \cos\tau \, d\tau$$

$$= \frac{1}{2}\int_{-\infty}^{t} e^{-t+\tau} \left(e^{i\tau} + e^{-i\tau}\right) d\tau$$

$$= \frac{1}{2} e^{-t} \int_{-\infty}^{t} e^{(1+i)\tau} + e^{(1-i)\tau}\, d\tau$$

$$= \frac{1}{2} e^{-t} \left[\frac{1}{1+i} e^{(1+i)t} + \frac{1}{1-i} e^{(1-i)t} \right]$$

$$= \frac{1}{2} \left[\frac{1-i}{2} e^{it} + \frac{1+i}{2} e^{-it} \right]$$

$$= \frac{1}{2}\cos t + \frac{1}{2}\sin t$$

3)

$$\hat{f}(\omega) = \frac{1}{1+i\omega}$$

$$\hat{g}(\omega) = \pi\,\delta(\omega-1) + \pi\,\delta(\omega+1)$$

$$\hat{f}(\omega)\,\hat{g}(\omega) = \frac{\pi}{1+i\omega}\,\delta(\omega-1) + \frac{\pi}{1+i\omega}\,\delta(\omega+1)$$

$$= \frac{\pi}{1+i}\,\delta(\omega-1) + \frac{\pi}{1-i}\,\delta(\omega+1)$$

$$(f*g)(t) = \frac{1}{2(1+i)}\,e^{it} + \frac{1}{2(1-i)}\,e^{-it}$$

$$= \frac{1}{2}\cos t + \frac{1}{2}\sin t$$

4)

a) $Ax = 0$ als $x_1 = -x_2$

$x_3 = -x_4$

$x_5 = -x_6$

$\ker A = \{ x \in \ell_2 \mid x_{2k} = -x_{2k-1} \text{ voor } k = 1, 2, \ldots \}$

b) We hebben:

$$\|y\|^2 = (x_1 + x_2)^2 + (x_3 + x_4)^2 + \ldots$$
$$\leq (2x_1^2 + 2x_2^2) + (2x_3^2 + 2x_4^2) + \ldots$$
$$= 2\|x\|^2$$

Bovendien als $x = (1, 1, 0, 0, \ldots)$

dan $y = (2, 0, 0, 0, \ldots)$

en $\|y\|^2 = 4 = 2\|x\|^2$

dus $\|A\| = \sqrt{2}$

5 a)

$$H(s) = \frac{s+1}{s^2-3s+2}$$

$$= \frac{s+1}{(s-2)(s-1)}$$

$$= \frac{A}{s-2} + \frac{B}{s-1}$$

$$A(s-1) + B(s-2) = s+1$$

$s=1$: $-B=2$ $\quad B=-2$

$s=2$: $A=3$

$$H(s) = \frac{3}{s-2} - \frac{2}{s-1}$$

$$h(t) = 3e^{2t}\mathbb{1}(t) - 2e^{t}\mathbb{1}(t)$$

b)

$$g(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$= \tfrac{3}{2}(e^{2t}-1)\mathbb{1}(t) - 2(e^{t}-1)\mathbb{1}(t)$$

$$= \left(\tfrac{3}{2}e^{2t} - 2e^{t} + \tfrac{1}{2}\right)\mathbb{1}(t)$$

5 c)

$$u(t) = \cosh t$$

$$U(s) = \frac{1}{2}\frac{1}{s-1} + \frac{1}{2}\frac{1}{s+1}$$

$$u'(t) \longleftrightarrow s\,U(s) - 1$$

$$y(t) \longleftrightarrow Y(s)$$

$$y'(t) \longleftrightarrow s\,Y(s) - 1$$

$$y''(t) \longleftrightarrow s^2\,Y(s) - s + 1$$

$$(s^2 - 3s + 2)\,Y(s) - s + 1 + 3 = (s+1)\,U(s) - 1$$

$$Y(s) = \frac{s+1}{s^2-3s+2}\,U(s) + \frac{s-5}{s^2-3s+2}$$

$$= \frac{1}{2}\frac{s+1+s-1}{(s-1)(s^2-3s+2)} + \frac{(s-5)(s-1)}{(s-1)(s^2-3s+2)}$$

$$= \frac{s^2-5s+5}{(s-1)^2(s-2)}$$

$$= \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s-2}$$

$$A(s-1)(s-2) + B(s-2) + C(s-1)^2 = s^2 - 5s + 5$$

$s = 1: \quad -B = 1 \qquad B = -1$

$s = 2: \quad C = -1$

$s^2 \quad : \quad A + C = 1 \qquad A = 2$

$$Y(s) = \frac{2}{s-1} - \frac{1}{(s-1)^2} - \frac{1}{s-2}$$

$$y(t) = 2e^t - te^t - e^{2t} \quad \text{voor } t > 0$$

Note

$$u(t) + u'(t) \Longleftrightarrow (s+1)\,U(s) - 1$$

$$= \frac{1}{s-1}$$