Course : **Graph Theory**
Date : 21 April 2017
Time : 13.45-16.45 uur

**Motivate all your answers. The use of electronic devices is not allowed.**

**In this exam a graph $G$ is a *simple* graph, i.e, $G$ has no loops and two distinct vertices are connected by at most one edge.**

1. [5 pt]
   Let $v$ be a cut vertex of $G$.
   Prove that $G^c - v$ is connected ($G^c$ is the complement of $G$).

2. [5 pt]
   $T$ is a tree. Prove that:
   all vertices of $T$ have odd degree if and only if
   for each $e \in E(T)$, both components of $T - e$ are odd (an odd component is a component with an odd number of vertices).

3. [6 pt]
   Prove that $\kappa(Q_k) = k$, i.e, the $k$-cube is $k$-connected.

4. [5 pt]
   $M$ is a maximum matching in a bipartite graph $G$. Prove that: $|M| \geq \dfrac{\epsilon(G)}{\Delta(G)}$.

5. (a) [2 pt] $G$ is a graph with $\nu = 2n + 1$ and $\varepsilon > n\Delta$. Prove that $\chi' = \Delta + 1$.
   (b) [3 pt] $G$ is a graph which is obtained from a $k$-regular graph with an odd number of vertices, by deleting fewer than $\dfrac{k}{2}$ edges.
   Prove that $\chi' = \Delta + 1$.

6. [5 pt]
   Let $G$ be a graph with the property that each pair of odd cycles in $G$ have a vertex in common. Prove that $\chi(G) \leq 5$.

7. [5 pt]
   Let $G$ be a bipartite planar graph. Prove that $\delta(G) \leq 3$.

**Total:** 36 + 4 = 40 points