Exam for LNMB Course on Scheduling 2012

This exam consists of:

- **2 pages.**
- **5 questions.**
- You can obtain a total of 50 points. Your exam grade will be the points you obtained divided by 5.

When a proof is asked, please provide a mathematically sound proof, short but precise. Unless stated otherwise, you are always expected to (briefly) explain your answer.
In case the objective function is not explicitely specified, it is supposed to be a regular objective function, unless stated otherwise.

Exercise 1 (10 points)

(a) Consider the problem $\mathrm{P}|p_j = 1, \mathrm{prec}|C_{\max}$. Show that deciding whether $C_{\max} \leq 3$ is NP-complete by a reduction from Clique.

Hint: For each timeslot $[0, 1)$, $[1, 2)$, and $[2, 3)$ create one or more dummy job such that all dummy jobs scheduled in the first time slot preceed all dummy jobs scheduled in the second time slot and all dummy jobs for the second preceed all dummy jobs for the third time slot.

Furthermore, transform the graph into jobs with precedence constraints and find the right number of machines such that all jobs can be scheduled in 3 time units if and only if a clique of size at least K exists.

(b) What is a lower bound on the performance guarantee of any polynomial-time approximation algorithm, assuming that $\mathrm{P} \neq \mathrm{NP}$.

A definition of Clique-problem is given at the end of this exam.

Exercise 2 (10 points)

Consider the problem $1|r_j|L_{\max}$ such that whenever $r_j < r_k$, then also $d_j < d_k$. Show that the Earliest Due Date (EDD) rule solves this problem to optimality.

Exercise 3 (10 points)

(a) Show that $\mathrm{F}|\mathrm{pmtn}|C_{\max}$ is equivalent to $\mathrm{F}||C_{\max}$.

(b) Is this also true for $\mathrm{O}|\mathrm{ptmn}|C_{\max}$ and $\mathrm{O}||C_{\max}$?

Exercise 4 (10 points)
Consider the problem $P|r_j|C_{\text{max}}$. A list-scheduling algorithm for this problem works as follows: whenever a machine becomes idle, it starts processing an available job. If more than two jobs are available, ties are broken according to their position in a predefined list of the jobs.

Show that this algorithm has a performance guarantee of $(2 - 1/m)$.

Exercise 5 (10 points)
Consider the stochastic scheduling problem $P||\mathbb{E}\,[\,C_{\text{max}}\,]$ with $n = 3$ jobs which are independently distributed according to the following distributions:

- Job 1 has deterministic size $p_1 = 2$.
- $\Pr[P_2 = 1] = \Pr[P_2 = 3] = 1/2$
- Job 3 has processing time from a uniform distribution over $[0, 2]$.

(a) Given an optimal policy for $m = 1$.

(b) Given an optimal policy for $m = 2$.

(c) Given an optimal policy for $m = 3$.

Definition 1 (Clique) *Given a graph $G = (V, E)$ and an integer K. Does G contain a clique of size at least K, i.e., a subset $V' \subseteq V$ of cardinality $|V'| \geq K$ such that for any two vertices $u, v \in V'$, $\{u, v\} \in E$.*

Clique is known to be strongly NP-hard.