Course 191551200 “Scientific Computing”
test T_1

February 22, 2012, 13:45–14:05

Your name: _____

Your student number: _____

Space for your drafts (will not be checked)

1. (25 points[1]) Determine $L \in \mathbb{R}^{3\times 3}$ such that $L^{-1} = L_2 L_1$,

$$L_1 = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}, \quad a, b, c \in \mathbb{R}.$$

   You are supposed to write down L, rather than to compute L.

2. (10 p) Give the definition of the Schur decomposition.

3. (25 p) Determine the Schur decomposition of the matrix A (given below) by finding suitable permutations of rows and columns.

$$A = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}, \quad a, b, c, d, e, f \in \mathbb{R}.$$

[1] Total number of points is 100.

Space for your drafts (will not be checked)

4. (10 p) Give the definition of the SVD of a matrix $A \in \mathbb{C}^{m\times n}$, $m \geqslant n$.

5. (30 p) Let $A \in \mathbb{R}^{3\times 2}$. It is known that if a vector $x \in \mathbb{R}^2$ is written as $x = \alpha \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, with $\alpha, \beta \in \mathbb{R}$, then

$$Ax = 2\alpha \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Determine the SVD of A. Motivate your answer.