Course 19.155120.0 "Scientific Computing"
test T_2

April 24, 2013, 13:45–14:05

Your name: _____

Your student number: _____

Space for your drafts (will not be checked)

Question 1 (40 points) A linear system $Ax = b$ is solved, with $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$ given.

(a) (10 p) Write down the right-preconditioned system $\tilde{A}\tilde{x} = \tilde{b}$ for a preconditioner matrix $M \in \mathbb{R}^{n \times n}$—more precisely, specify $\tilde{A}$, $\tilde{x}$, $\tilde{b}$ in terms of A, x, b and M.

(b) (15 p) Write down the (unpreconditioned) Richardson method for solving the right-preconditioned system given above. After that rewrite the Richardson method in terms of A, x, b and M.

(c) (15 p) For which choice of M will the Richardson iteration converge in the fastest possible way? Motivate your answer.

Space for your drafts (will not be checked)

Question 2 (30 points) For a matrix $A \in \mathbb{R}^{n \times n}$ it is known that all its Rayleigh quotients lie on the line $2 + i\beta$ in the complex plane, with $\beta \in \mathbb{R}$ and $i^2 = -1$. The line is thus parallel to the imaginary axis and crosses the real axis at point $2 + i0 = 2$. Is it true that the Ritz values of A will also lie on the line $2 + i\beta$? Why or why not?

Question 3 (30 points) Write down the implicit trapezoidal rule for the initial value problem $w'(t) = -Aw(t) + g(t)$, $w(0) = w^0$. After that rewrite the scheme as a linear system where the unknown vector is the solution w^{k+1} at the next time level, i.e., $w^{k+1} \approx w(\tau(k+1))$, with $\tau > 0$ being the time step size and k the time step index.