Course 19.155120.0 “Scientific Computing” test T_2

reexamination

May 6, 2014, 13:05–13:30

Your name: ________________________________

Your student number: ________________________________

Space for your drafts (will not be checked)

Q1 A linear system $Ax = b$, with $A \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^n$ given, is solved by a Krylov subspace method.

(10 p) (a) Finish the definition: an iterative method $x_0,\ x_1,\ \ldots,\ x_k,\ \ldots$ is called a Krylov subspace method if

$$x_k = x_0 + \ldots\ldots, \qquad \in \mathcal{K}(\ldots\ldots, \ldots\ldots),$$

where $\mathcal{K}(\ldots\ldots, \ldots\ldots)$ is called .... and is defined as

$$\mathcal{K}(\ldots\ldots, \ldots\ldots) = \ldots$$

(20 p) (b) Define the residual vector r_k. Show that $x_k - x_0$ is a polynomial in A (called the update polynomial) times the initial residual vector. Define the residual polinomial P_k and determine $P_k(0)$.

(20 p) (c) Assume we use the following basis of the Krylov subspace:

$$v_1 = r_0,\ v_2 = Ar_0,\ v_3 = A^2 r_0,\ \ldots,\ v_k = A^{k-1} r_0$$

and the basis vectors v_i are the columns of matrix V_k. For a certain Krylov subspace method $x_k = x_0 + V_k\bar{y}$, $V_k \in \mathbb{R}^{n\times k}$, $\bar{y} \in \mathbb{R}^k$. Express the coefficients of the update polynomial $x_k - x_0$ in terms of $\bar{y}$.

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(20 p) Q2 For given large, sparse $A, B \in \mathbb{R}^{n\times n}$, consider generalized eigenproblem

$$Ax = \lambda Bx.$$

B^{-1} can not be computed because it does fit into the computer memory. However linear systems $Bu = v$ can be solved for any $v \in \mathbb{R}^n$. Can the given eigenproblem be solved with the Arnoldi method? If no, explain why. If yes, explain how you would organize computations.

(30 p) Q3 Give a definition of the stability region of a Runge-Kutta method. Determine the stability region of the explicit Euler scheme, assuming that the λ appearing in the stability test problem are the eigenvalues of a skew-symmetric matrix A. Would you recommend using explicit Euler for solving $y' = Ay$?