Course 19.155120.0 "Scientific Computing"
test T_3

June 5, 2013, 14:45–15:05

Your name: _____

Your student number: _____

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Question 1 (35 points) Let $f(x)$ be a sufficiently smooth function acting from a domain $\Omega \subset \mathbb{R}^n$ to $\mathbb{R}$, $f : \Omega \to \mathbb{R}$. Let $x_* \in \Omega$.

Q1a (15 p) Write down the first several terms of the Taylor series of the function $f(x)$ around x_*, ending with the term $O(\|x - x_*\|^2)$.

Q1b (20 p) Assume the Hessian $f''(x_*)$ is negative definite and $\nabla f(x_*) = 0$. Is x_* a stationary point of f? Is x_* an optimal point of f? If x_* is an optimal point, is it a maximum or a minimum point? Motivate your answer.

Question 2 (50 points) A nonlinear system of equation $F(x) = 0$ is solved by an inexact Newton method, where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth mapping.

Q2a (15 p) Fill in the missing part in the formula of the Newton method below.

$$x_{k+1} = x_k + \ldots\ldots$$

Q2b (15 p) The linear system in the Newton method is solved by an iterative solver inexactly, with some residual $r \in \mathbb{R}^n$. Write down the expression for the residual. After that rewrite this expression as a perturbed linear system. Note that the approximate solution provided by the iterative solver is the *exact* solution of this perturbed system.

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Q2c (10 p) Complete the formula for the inexact Newton method described in the previous question.

$$x_{k+1} = x_k + \ldots\ldots$$

Q2d (10 p) It is known that the Jacobian of F is a symmetric matrix. Which iterative solver would you suggest for solving the linear system at each Newton iteration? Why?

Question 3 (15 points) Let A, B and X be real matrices such that the product AXB can be computed. Is it possible to write

$$\mathsf{vec}(AXB) = \mathcal{M}\,\mathsf{vec}(X)$$

for some matrix $\mathcal{M}$? If yes, derive an expression for $\mathcal{M}$.