Time Series Analysis (& SI)—191571090

(The first 1.5 hours it is CLOSED BOOK. The second 1.5 hours is OPEN BOOK.)

Date: 4-nov-2016
Place: Sports center
Time: 08:45–11:45

1. **CLOSED BOOK:** Three questions:

   (a) Let $r(0) = 3, r(\pm 1) = 2, r(\pm 2) = 1$ and all other $r(k) = 0$. Is this a covariance function?

   (b) When do we say that an estimator is "efficient"?

   (c) Is it correct to claim that U_t defined as $U_t = B(\mathrm{q})\epsilon_t$ is in general not a good choice as input for system identification if $B(z)$ has a zero on the unit circle (i.e. $B(z_1) = 0$ for some $z_1 \in \mathbb{C}$ with $|z_1| = 1$)?

2. **CLOSED BOOK**: Consider the ARMA process

   $$X_t = aX_{t-2} + \epsilon_t + b\epsilon_{t-1}.$$

   (Notice the "$t-2$".)

   (a) For which a, b is X_t asymptotically wide sense stationary?

   (b) For which a, b is the scheme invertible.

   (c) Assume it is asymptotically WSS and invertible. Determine the 1-step ahead predictor

   (d) Assume it is asymptotically WSS and invertible. Determine the 2-step ahead predictor

   (e) Actually the standard formula for the 1-step ahead predictor assumes that ϵ_t is zero mean. How would you modify the above 1-step ahead predictor if $\mu := \mathbb{E}\epsilon_t$ is nonzero?

   (f) Determine all a, b for which X_t is white noise.

3. **CLOSED BOOK:** The Gauss-Markov theorem says that the best linear unbiased estimator (BLUE) $\hat{\theta} = KX$ of θ where

   $$X = W\theta + \epsilon,$$

   equals the least squares solution if ϵ is zero mean and has covariance matrix $\sigma^2 I$. What is the BLUE $\hat{\theta} = KX$ in case ϵ is zero mean but has a diagonal covariance matrix of the form

   $$R_\epsilon = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \ddots \end{bmatrix}.$$

4. **CLOSED BOOK:** Given data $x_0, \ldots, x_{N-1}$ is it numerically easier to estimate the parameters of an AR-model or of an MA-model?

5. **OPEN BOOK:** In § 6.1 it is proved that the variance of the sample mean $\hat{m}_N = \frac{1}{N}(x_0 + \cdots + x_{N-1})$ for wide-sense stationary processes X_t is

$$\frac{1}{N} \sum_{k=-N+1}^{N-1} (1 - |k|/N) r(k).$$

The problem, of course, is that we do not know $r(k)$. Some researchers suggest to estimate this variance as

$$\frac{1}{N} \sum_{k=-M}^{M} (1 - |k|/N) \hat{r}_N(k)$$

for some suitable choice of M. How would you choose M? (Explain your choice of M.)

6. **OPEN BOOK:** Suppose X_t is zero mean white noise. The periodogram $p_N(\omega)$ is defined as

$$P_N(\omega) = \frac{1}{N} \left| \sum_{t=0}^{N-1} X_t \mathrm{e}^{-\mathrm{i}\omega t} \right|^2.$$

For each ω compute $\mathbb{E}(P_N(\omega))$.

7. **OPEN BOOK:** In the lecture notes the system h_t is resolved from Equation (8.10) through Fourier transformation, but it can also be done in time domain if we assume that the system has "finite impulse response" meaning that

$$y_t = \sum_{m=0}^{m=M} h_m u_{t-m} + v_t$$

for some finite $M > 0$. That is, $h_t = 0$ for all $t > M$.

(a) Show that the Equation (8.10) for finite impulse response systems implies that

$$\begin{bmatrix} r_u(0) & r_u(1) & \cdots & r_u(M) \\ r_u(1) & r_u(0) & \cdots & r_u(M-1) \\ \vdots & \vdots & \vdots & \vdots \\ r_u(M) & r_u(M-1) & \cdots & r_u(0) \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_M \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ \vdots \\ ? \end{bmatrix}$$

for certain numbers "?" and determine all these "?"

(b) Chapter 8 makes the point that u_t should be "sufficiently rich" (final part of § 8.1.3). Can you argue from the claim of Problem 3.10(c) of the lecture notes—page 26—that sufficiently rich ensures that the above equation has a unique solution $h_0, \ldots, h_M$?

problem:	1	2	3	4	5	6	7
points:	3+2+2	2+1+3+2+2+2	3	1	4	4	3+2

Exam grade is $1 + 9p/p_{\max}$.