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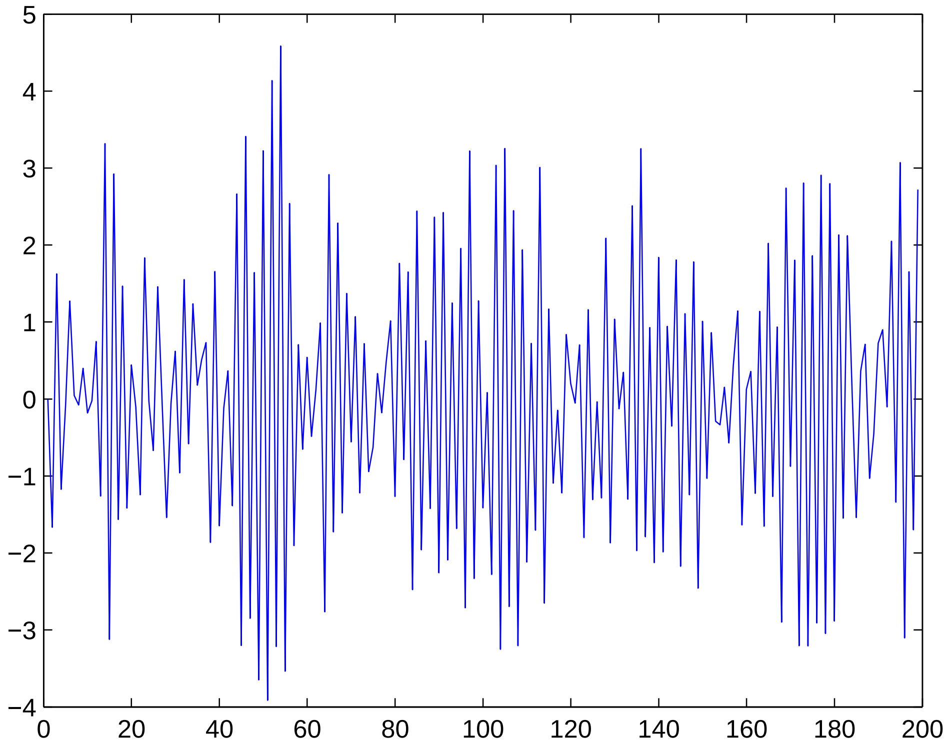
| Course | : | **Time Series Analysis** |
| Code | : | 157109 |
| Date | : | Friday, November 9, 2007 |
| Time | : | 09.00–12.00 |
| Place | : | WA-220 |

**All answers must be motivated.**




1. The above plot depicts a realization $x_0, \ldots, x_{200}$ of a stochastic process $X_t$. It was generated by *one* of the three schemes below with zero mean, unit variance white noise $\varepsilon_t$:

   $$X_t = 0.9X_{t-1} + \varepsilon_t, \qquad X_t = -0.9X_{t-1} + \varepsilon_t, \qquad X_t = -0.2X_{t-1} + 10\varepsilon_t.$$

   Explain which one the three was used and why the other two are unlikely.

2. Consider the following system

   $$3X_t + X_{t-1} = 3\varepsilon_t - \varepsilon_{t-1} \tag{1}$$

   with $\varepsilon_t$ white noise.

   (a) Is this system stable?

   (b) Is this system invertible?

   (c) Is the system wide-sense stationary?

   (d) Determine its spectral density.

3. Suppose $X_1, \ldots, X_N$ are mutually independent stochastic variables with the same probability density function

$$f(x) = \begin{cases} \frac{4x}{\lambda^2} \exp(-2x/\lambda), & \text{if } x > 0 \\ 0 & \text{if } x < 0. \end{cases} \tag{2}$$

It is known that $\mathbb{E}X_t = \lambda$, and $\mathbb{E}X_t^2 = \frac{3}{2}\lambda^2$.

(a) Show that $\operatorname{var}(X_t) = \frac{\lambda^2}{2}$.

(b) Give the joint probability density of $X_1, X_2, \ldots, X_N$.

(c) Determine the maximum likelihood estimator $\hat{\lambda}$ of $\lambda$ given $X_1, \ldots, X_N$.

(d) Is the estimator $\hat{\lambda}$ unbiased?

(e) Is the estimator $\hat{\lambda}$ efficient?

4. In Equation (5.65) the dynamics for the one-step predictor is derived under the assumption that the coefficient $b_0$ of the polynomial $N(q)$ is one. Find the dynamics of the one-step predictor if this assumption is not satisfied.

Note that $h_0$ will not be equal to one, but you may assume that all the other assumptions are satisfied.

5. Consider the independent stochastic variables $X_1, \ldots, X_N$ with the same probability density function

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } x \in [\lambda - 1, \lambda + 1] \\ 0 & \text{elsewhere.} \end{cases} \tag{3}$$

We want to estimate the parameter $\lambda$ of this process.

(a) Calculate $\mathbb{E}X_t$.

(b) Show that $\operatorname{var}(X_t) = \frac{1}{3}$.

(c) As estimator for $\lambda$, we use

$$\hat{\lambda}_N = \frac{1}{N} \sum_{t=1}^{N} X_t. \tag{4}$$

Explain this choice.

(d) Is this estimator biased?

(e) Calculate the variance of this estimator.

(f) Is the estimator consistent?

**Normering:**

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|---|---|---|---|---|---|---|---|
| 1 : 10 | 2 | a : 5 | 3 | a : 5 | 4 : 10 | 5 | a : 5 |
| | | b : 5 | | b : 2 | | | b : 3 |
| | | c : 5 | | c : 6 | | | c : 3 |
| | | d : 5 | | d : 6 | | | d : 4 |
| | | | | e : 6 | | | e : 6 |
| | | | | | | | f : 4 |

**Total:** 90 + 10 = 100 points