UNIVERSITEIT TWENTE
Faculteit Elektrotechniek, Wiskunde en Informatica

Exam Random Signals and Filtering (201200135) on Monday April 15, 2013, 13.45 – 16.45 hours.

The solutions of the exercises need to be clearly formulated and written in a well-structured manner. Moreover, you always need to present a derivation or arguments to support your answer.

You can use one single-sided A4 page of handwritten notes with your exam.

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1. Consider $\Omega = [0, 1]$ and let $\mathcal{P}$ be such that

$$\mathcal{P}([a, b]) = (b - a)^2$$

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for $0 \leq a < b \leq 1$. Verify whether $\mathcal{P}$ satisfies all axioms of a probability measure.

2. Consider the following linear system:

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$$\begin{aligned} X_{k+1} &= \sqrt{X_k} + W_k \\ Y_k &= \sqrt{X_k} + V_k \end{aligned}$$

where X_0, V_k and W_k are mutually independent and all have a uniform distribution on the interval $[0, 1]$. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

a) Determine the density function associated to the stochastic variable X_1

b) Determine $E[X_0 | Y_0]$

c) Determine $E[X_1 | Y_0]$.

3. Consider the following linear system:

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$$\begin{aligned} X_{k+1} &= X_k^2 + W_k \\ Y_k &= X_k + V_k \end{aligned}$$

where X_0, V_k and W_k are mutually independent and all have a Gaussian distribution with mean 0 and variance 1. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

Determine an estimate of X_1 given measurements Y_0 and Y_1 using the extended Kalman filter (the Kalman filter based on linearization).

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4. Consider the following linear system:

$$\begin{aligned} X_{k+1} &= X_k + W_k \\ Y_k &= X_k + V_k \end{aligned}$$

where X_0, V_k and W_k are mutually independent and all have a Gaussian distribution with mean zero and variance 1. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

We are applying a particle filter without resampling where we recursively update our particles according to:

$$\pi(x_k \mid x_{k-1}^i, y_k) = p(x_k \mid x_{k-1}^i, y_k)$$

a) Clarify how you could implement the updating of particles in a program such as Matlab (the algorithm not the precise Matlab code)

b) Compute $\operatorname{var}(X_k^i - X_k)$ as a function of $\operatorname{var}(X_{k-1}^i - X_{k-1})$.

c) Argue whether or not a particle filter without resampling might work well in this case.

You can earn the following number of points for each exercise:

Exercise	1.	2 points	Exercise	2.	5 points
Exercise	3 ~~4~~.	5 points	Exercise	4 ~~5~~.	6 points

The grade is determined by adding two points to the total number of points and dividing by two.