Solution/Correction standard, 2nd Test Mathematics B1; October 25, 2013.

1. (a) [1 pt] Show that the dot product of $\mathbf{u}$ and $\mathbf{v}$ is 0. **[.5 pt]**

$$\mathbf{u} \cdot \mathbf{v} = \langle 1, -1, -1\rangle \cdot \langle -1, 2, -3\rangle = -1 - 2 + 3 = 0.$$ **[.5 pt]**

(b) [1 pt]

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & -3 \end{bmatrix} \begin{matrix} 1 & -1 \\ -1 & 2 \end{matrix}$$

$$\mathbf{w} = \mathbf{u} \times \mathbf{v} = \langle 5, 4, 1\rangle.$$ **[1 pt]**

(c) [1 pt] Two alternatives:

(1) Vector $\mathbf{v}$ is a solution:
From (a) follows: $\mathbf{v}$ is orthogonal to $\mathbf{u}$.
From (b) follows: $\mathbf{v}$ is orthogonal to $\mathbf{w}$. **[1 pt]**

Or:

(2) Calculate $\mathbf{u} \times \mathbf{w}$:

$$\begin{bmatrix} 1 & -1 & -1 \\ 5 & 4 & 1 \end{bmatrix} \begin{matrix} 1 & -1 \\ 5 & 4 \end{matrix}$$

$$\mathbf{u} \times \mathbf{w} = \langle 3, -6, 9\rangle.$$ **[1 pt]**

2. (a) [2 pt] Rewrite z:

$$z = \frac{\sqrt{2}}{1-i} = \frac{\sqrt{2}}{1-i} \cdot \frac{1+i}{1+i} = \frac{\sqrt{2}(1+i)}{2} = \tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}\sqrt{2}\, i.$$

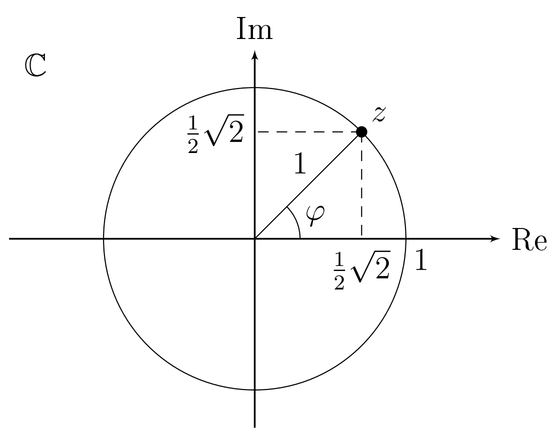
$$|z|^2 = \left(\tfrac{1}{2}\sqrt{2}\right)^2 + \left(\tfrac{1}{2}\sqrt{2}\right)^2 = 1$$ **[1 pt]**

For the argument φ of z two methods can be used:

$$\tan\varphi = \frac{\frac{1}{2}\sqrt{2}}{\frac{1}{2}\sqrt{2}} = 1,$$

and z is in the right-half plane, hence $\varphi = \dfrac{\pi}{4}$. **[1 pt]**

Alternatively, use a picture:



[1 pt]

(b) [2 pt] From (a) follows: $z = e^{\frac{1}{4}\pi i}$ hence $z^6 = e^{\frac{6}{4}\pi i} = e^{\frac{3}{2}\pi i} = -i$. **[1 pt]**

Therefore $\mathrm{Re}\, z^6 = 0$ and $\mathrm{Im}\, z^6 = -1$. **[1 pt]**

3. (a) [3 pt] Set up the characteristic equation:

$$\lambda^2 + 4\lambda + 4 = 0.$$ **[.5 pt]**

This equation has one real solution:

$$\lambda = -2.$$ **[.5 pt]**

The general solution of the differential equation then is

$$c_1 e^{-2t} + c_2 t e^{-2t}.$$ **[1 pt]**

Finally, use the initial conditions to determine c_1 and c_2.
From $y(0) = .2$ follows $c_1 = 0.2$ **[.5 pt]**
Differentiate y:

$$y'(t) = -0.4e^{-2t} + c_2(1 - 2t)e^{-2t}.$$

From $y'(0) = -1.2$ follows $c_2 = -0.8$ **[.5 pt]**

(b) [2 pt] From (a) follows

$$y(t) = 0.2e^{-2t} - 0.8te^{-2t}.$$

We look for t_0 where $y(t_0) = 0$, in other words: t_0 is the root of

$$0.2e^{-2t} - 0.8te^{-2t} = 0.$$ **[1 pt]**

Rewrite the equation:

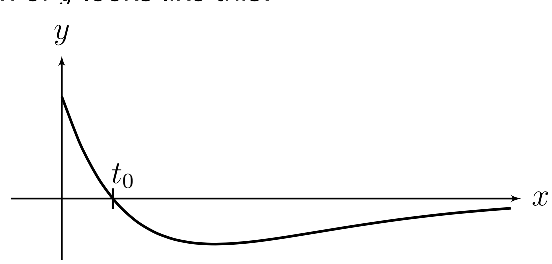
$$(0.2 - 0.8t)e^{-2t} = 0.$$

Since e^{-2x} is not equal to zero we have

$$0.2 - 0.8t = 0,$$

which gives $t_0 = \dfrac{1}{4}$. **[1 pt]**

Note: the graph of y looks like this:



Total: 12 points