
ATTENTION: Problems 1–4 need to be solved only by students doing the *full* exam (i.e. both the "E" and the "M" parts, not just "M").
Problems 5 – 8 need to be solved by *all* students.
Problems 9 & 10 need to be solved only by students that take just the "M"-test (i.e. NOT by the students that do the full exam).
Students taking the full "E+M" exam have 3 hours and are graded on a maximum of 150 points.
Students taking just the "M" test have 2 hours and are graded on a maximum of 100 points.

Problem 1 (E+M, 10 points). Consider a parallel-plate capacitor with plate size >> inter-plate distance *d*, so that it can be considered as infinitely extended. Inside the capacitor, each metal plate is coated with a dielectric layer of thickness *d*/3 and relative dielectric constant $\varepsilon_r = 2$ (Figure 1).

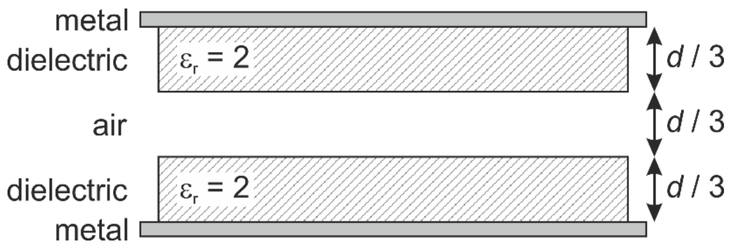

Figure 1: parallel-plate capacitor symmetrically filled for 2/3 of its volume (Problem 1).

Copy the figure and sketch all free charge σ_f ; all bound charge σ_b ; and the electric field lines E (clearly showing both direction and magnitude of the field).

Problem 2 (E+M, 15 points). Below you find five statements. For each of them, indicate whether the statement is 'true' (T) or 'not true' (NT). Also include a brief argument why you agree or not (minimum 1 & maximum 5 lines per statement). Read the statements carefully, each word may be important!

2.a. Inside a non-conducting sphere that carries a homogeneous volumetric charge density ρ_0, the strength |E| of the electric field generated by this charge is constant.

2.b. A long and thin-walled metal cylinder with radius *R* carries a homogeneous surface charge density σ. The E field outside this cylinder is the same as the field that would be generated by a long wire along the cylinder's axis that carries a uniform line charge density $\lambda = 2\pi R\sigma$.

2.c. If $\hat{\mathbf{n}}$ is a *normal* unit vector somewhere on an equipotential surface and E is the electric field at that location, then $\mathbf{E} \times \hat{\mathbf{n}} = 0$.

2.d. When one pulls the plates of a charged and disconnected capacitor apart, the energy stored in the capacitor increases.

2.e. A solid sphere is polarized such that $\mathbf{P} = \mathrm{k}r\hat{\mathbf{r}}$ with k a constant, *r* the radial distance to its centre and $\hat{\mathbf{r}}$ the usual spherical radial unit vector. The corresponding bound charge density is $\rho_b = \mathrm{k}$.

Problem 3 (E+M, 25 points).

3.a. Show that the electric field E that a straight piece of homogeneously charged wire with finite length *L* generates in a point P at a perpendicular distance *R* of the wire can be written as

$$E_{\perp} = \frac{\lambda}{4\pi\varepsilon_0}\frac{\sin\theta_2 - \sin\theta_1}{R}$$

$$E_{//} = \frac{\lambda}{4\pi\varepsilon_0}\frac{\cos\theta_2 - \cos\theta_1}{R}$$

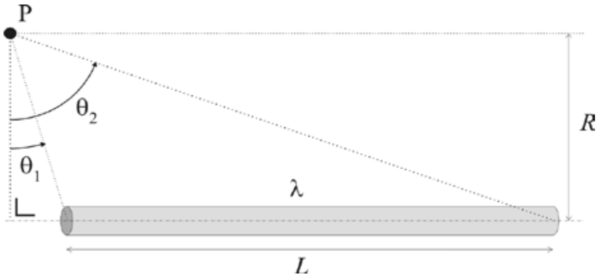

Figure 2: a homogeneously charged straight piece of wire (Problem 3).

λ is the homogeneous charge density, $E_{\perp}$ and $E_{//}$ are the field components perpendicular and parallel to the wire, θ_1 and θ_2 are the angles that the "sight-lines" from *P* to the ends of the wire make with direction perpendicular to the wire (Figure 2). (*hint: the integration required to solve this problem is easiest with θ as integration variable*).

3.b. Work out the field strength when the point P is equidistant to both ends of the wire. Express your answer in terms of λ , *L* en *R*.

3.c. Work out the limit of answer 3.b when L >> R, in other words: using 3.b work out the field of an infinitely long wire.

Problem 4 (E+M, 25 points). A large slab of dielectric material has a flat surface. Homogeneously spread out over this surface, there's a *free* charge density σ_f . In the air ($\varepsilon = \varepsilon_0$) just above the dielectric slab, there's a uniform electric field E_{air} with field strength | E_{air} | = 1 N/C. The field lines E_{air} make an angle of 60° with the direction normal to the surface.

Inside the dielectric, the electric field E_{slab} is also uniform. E_{slab} makes an angle of 45° with the normal to the surface. The relative dielectric constant of the slab is $\varepsilon_r = 2$.

4.a. Which component of E needs to be continuous at the surface?

4.b. Calculate E on both sides of the surface.

4.c. Calculate the total (free + bound) charge density on the surface.

4.d. Calculate the electric displacement D on both sides of the surface.

4.e. Calculate the free and bound charge density σ_f and σ_b on the surface separately.

Problem 5 (*all students*, 15 points). Two infinitely large flat plates are placed parallel to each other and carry the same homogeneous volumetric current density *J*, but each in opposite direction (Figure 3).

5.a. Through which of the six wire loops A – F the magnetic flux is NOT zero?

5.b. The plates are pulled apart while the current density *J* is kept constant. What happens with flux through the loop(s) from question 5.a ?

5.c. Does one need to do work to pull the plates apart, or does this release energy? Motivate your answer.

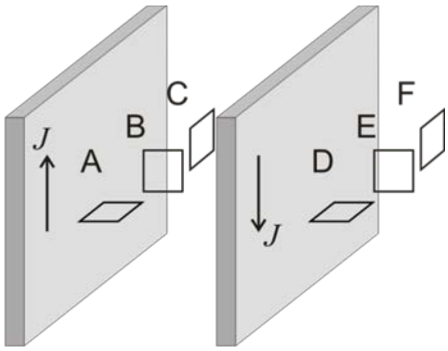

Figure 3: Parallel current-carrying plates (Problem 5).

Problem 6 (*all students*, 15 points). Below you find five statements. For each of them, indicate whether the statement is 'true' (T) or 'not true' (NT). Also include a brief argument why you agree or not (minimum 1 & maximum 5 lines per statement). Read the statements carefully, each word may be important!

6.a. In a part of space that only contains a homogeneous current density, the magnetic induction B is constant.

6.b. In the absence of free current the H-field is continuous crossing from one magnetic material to another, irrespective of its angle of incidence with the surface.

6.c. The total static magnetic flux through a closed surface is always equal to zero, even in the presence of free currents or magnetic materials.

6.d. Two circular conducting loops are placed co-axially above each other and in the bottom one a current is suddenly switched one. This causes the loops to repel each other.

6.e. A diamagnetic material placed outside, but near, the end of a short current-carrying coil is always pulled into this coil, irrespective of the direction of the current in the coil.

Problem 7 (*all students*, 25 points). Below, you're given four <u>static</u> vector fields F_i. a, b and c are constants.

$$\mathbf{F}_1(\mathbf{r}) = \mathrm{a}\,\hat{\mathbf{x}} + \mathrm{b}\,\hat{\mathbf{y}} + \mathrm{c}\,\hat{\mathbf{z}} \qquad \text{(Cartesian)}$$

$$\mathbf{F}_2(\mathbf{r}) = \mathrm{a}\,x\,\hat{\mathbf{x}} + \mathrm{b}\,\hat{\mathbf{y}} + \left(\frac{\mathrm{c}}{z}\right)\hat{\mathbf{z}} \qquad \text{(Cartesian)}$$

$$\mathbf{F}_3(\mathbf{r}) = \mathrm{a}\cos\phi\,\hat{\mathbf{s}} + \left(\mathrm{b} - \mathrm{a}\sin\phi\right)\hat{\boldsymbol{\varphi}} + \mathrm{c}\,\hat{\mathbf{z}} \qquad \text{(cylindrical)}$$

$$\mathbf{F}_4(\mathbf{r}) = \frac{\mathrm{a}}{r}\,\hat{\mathbf{r}} + \mathrm{b}\,\hat{\boldsymbol{\varphi}} \qquad \text{(spherical)}$$

7.a. For each of these fields, determine if they might be a magnetic induction B(r). If so, derive the current density J(r) that generates such a B-field.

7.b. For each of these fields, determine if they can represent a magnetic vector potential A(r). If so, once more describe the corresponding current density J(r).

Problem 8 (*all students*, 20 points). A square metal wire loop with side *a* moves with a constant velocity $\mathbf{v} = v\,\hat{\mathbf{x}}$ in the direction of one of its sides (Figure 4). In doing so, it moves from a region without magnetic field ($x < 0$, B = 0) into a region with a homogeneous field ($x > 0$, $\mathbf{B} = B\,\hat{\mathbf{z}}$).

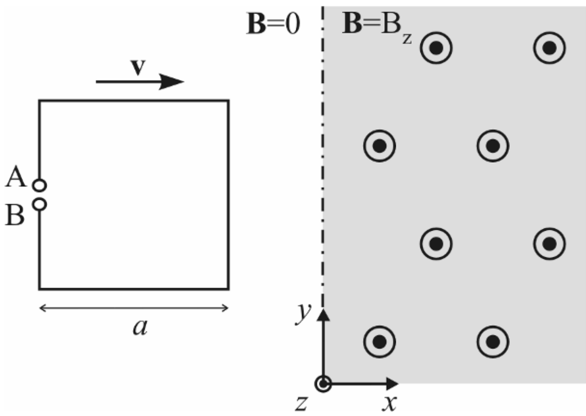

Figure 4: Moving wire loop (Problem 8).

8.a. The loop is interrupted between the points A and B. Sketch the induced e.m.f. $V_{AB}(t) = V_A - V_B$ during the loop's crossing of the *y*-axis as a function of the time *t* (choose $t = 0$ as the moment that the right-hand side of the loop reaches $x = 0$). Mind the sign of the voltage and quantify relevant magnitudes on the *V*- and *t*- axes in terms of *a*, *v* and *B*. The distance between A and B may be considered negligible with respect to the side *a*.

8.b. Repeat question 8.a., but this time for a double velocity $\mathbf{v} = 2v\,\hat{\mathbf{x}}$. In your sketch, use the same axes span as in 8.a. and once more quantify relevant magnitudes next to the axes.

8.c. Once more, repeat 8.a. but this time for a loop with double size (side 2*a*) and the original velocity ($\mathbf{v} = v\hat{\mathbf{x}}$). Also here, use the same axes span and quantify relevant magnitudes.

Problem 9 (only M, 10 points). A solid sphere of radius *R* consists of a permanent magnetic material with a 'frozen-in' uniform residual magnetization $\mathbf{M}_{\text{res}} = M_{\text{res}}\hat{\mathbf{z}}$ (Figure 5).

8.a. Draw a cross-section of the sphere in the *yz*-plane and sketch the magnetic field lines B inside and outside the sphere.

8.b. Derive a vector expression for the bound surface current K_b at any point on the surface of the sphere in terms of spherical coordinates (r, θ, ϕ).

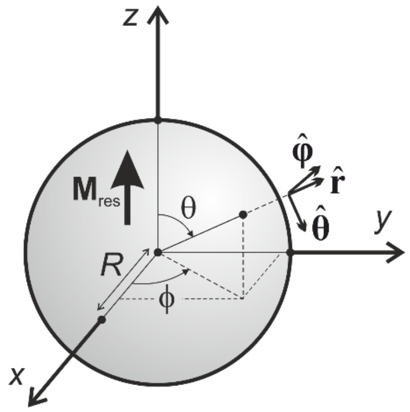

Figure 5: Spherical permanent magnet (Problem 9).

Problem 10 (only M, 15 points).

10.a. Two long cylinders (with radii *a* and *b*) are separated by a material with a uniform electrical conductivity σ (Figure 6). If they are maintained at a potential difference V, what current flows from one to the other, over a length *L*?

10.b. Now suppose that the conductivity of the material separating the cylinders is no longer uniform, but varies radially as $\sigma(s) = \frac{k}{s}$, with k a constant. Find the resistance between the cylinders.

(*hint: for any virtual coaxial cylindrical surface in-between* a *and* b, *the total current* I *crossing through that surface must be the same.*)

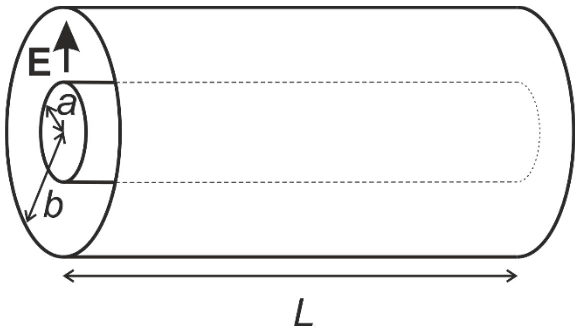

Figure 6: current flow between two cylinders (Problem 10).