Kenmerk: EWI2015/dmmp/013/BM

Exam Limits to Computing (201300042)

Thursday, October 29, 2015, 8:45 – 11:45

- Use of calculators, mobile phones, etc. is not allowed!
- This exam consists of four problems.
- Please start a new page for each problem.
- The total number of points is 63+7 = 70. The distribution of points is according to the following table.

1a: 6	2: 15	3a: 4	4a: 3
1b: 7		3b: 6	4b: 3
1c: 3		3c: 4	4c: 3
			4d: 3
			4e: 3
			4f: 3

1. Decidability and Recursive Enumerability

Consider the following problem:

$$\textsc{FixedPoint} = \{g \in \mathcal{G} \mid \exists n \in \mathbb{N} : \varphi_g(n) = n\}.$$

This means that FIXEDPOINT contains all Gödel numbers of WHILE programs that output n on input n for at least one number $n \in \mathbb{N}$.

(a) *(6 points)* Is FIXEDPOINT $\in$ REC? Prove your answer.

(b) *(7 points)* Is FIXEDPOINT $\in$ RE? Prove your answer.

(c) *(3 points)* Is FIXEDPOINT $\in$ co-RE? Prove your answer.

2. NP-Completeness

Let $G = (V, E)$ and $H = (U, F)$ be undirected graphs. We call H a subgraph of G if there exists an injective mapping $\pi : U \to V$ such that the following holds: For all $a, b \in U$ with $\{a, b\} \in F$, we have $\{\pi(a), \pi(b)\} \in E$.

Less formally: H is a subgraph of G, if G contains a copy of H. Or: H is a subgraph of G if we can remove nodes and edges of G to obtain a graph that is isomorphic to H.

Let

$$\textsc{SubGraph} = \big\{(G, H) \mid H \text{ is a subgraph of } G\big\}.$$

(15 points) Prove that $\textsc{SubGraph}$ is NP-complete.

Hint: $\textsc{Clique}$ is NP-hard.

3. Complexity Classes

Let $\mathsf{E} = \mathsf{DTime}\big(2^{O(n)}\big)$. Recall that $\mathsf{EXP} = \bigcup_{c>0} \mathsf{DTime}\big(2^{O(n^c)}\big)$.

(a) *(4 points)* Prove that $\mathsf{E} \subsetneq \mathsf{EXP}$.

(b) *(6 points)* Prove that E is not closed under polynomial-time many-one reductions. This means that there are problems A and B with $A \leq_{\mathsf{P}} B$ and $B \in \mathsf{E}$ and $A \notin \mathsf{E}$.

(c) *(4 points)* Prove that $\mathsf{E} \neq \mathsf{PSPACE}$.

Remark: Just prove that the two classes differ, you do not have to prove that $\mathsf{E} \subsetneq \mathsf{PSPACE}$ or $\mathsf{PSPACE} \subsetneq \mathsf{E}$ and, most likely, your proof does not answer if one class is a subset of the other.

4. Questions

Are the following statements true or false? Justify your answers.

(a) *(3 points)* If $\mathsf{P} = \mathsf{PSPACE}$, then $\mathsf{NP} = \mathsf{co\text{-}NP}$.

(b) *(3 points)* $3\mathrm{SAT} \in \mathsf{PSPACE}$.

(c) *(3 points)* If $3\text{-}\textsc{Coloring} \in \mathsf{NL}$, then $\mathsf{P} = \mathsf{NP}$.

(d) *(3 points)* For all $L \subseteq \mathbb{N}$, the following holds: If $L \leq H_0$, where H_0 denotes the special halting problem, then $L \notin \mathsf{co\text{-}RE}$.

(e) *(3 points)* For all $L \subseteq \mathbb{N}$, the following holds: If $L \in \mathsf{REC}$, then $\overline{L} = \mathbb{N} \setminus L \in \mathsf{RE}$.

(f) *(3 points)* If $\mathsf{NL} = \mathsf{NP}$, then $\mathsf{co\text{-}NP} \subsetneq \mathsf{PSPACE}$.