

WORK OUT ON A PAPER SEPARATE FROM LINEAR OPTIMIZATION

Module 2, test 1 Analysis, 201300057
13-12-2016,

Motivate all your answers.

1. Decide which of the following statements are true and which are false. Prove the true ones and give a counterexample for the false ones.
 - (a) (2pt) If B is a bounded subset of \mathbb{Q} , then $\inf(B) \in B$.
 - (b) (3pt) Let V, W be two nonempty subsets of \mathbb{R} that are bounded above. Then the set $V + W = \{v + w \mid v \in V, w \in W\}$ has a supremum.

2.
 - (a) (2pt) Formulate the Bolzano-Weierstrass-Theorem.
 - (b) (2pt) Prove or give a counterexample for the following statement:
If $\{x_n\}$ has a convergent subsequence, then $\{x_n\}$ is bounded.
 - (c) (3pt) Suppose that $\{x_n\}$ and $\{y_n\}$ are real sequences and both sequences are convergent. Prove

$$\lim_{n \rightarrow \infty} (x_n + y_n) = \lim_{n \rightarrow \infty} x_n + \lim_{n \rightarrow \infty} y_n.$$

3.
 - (a) (2pt) Give the definition of a Cauchy sequence.
 - (b) (4pt) Let $\{x_n\}$ be a sequence with the property

$$|x_n| \leq \frac{\sqrt{3n} + 2}{n + \sqrt{2}} \quad \text{for all } n \in \mathbb{N}$$

Determine if this sequence is Cauchy and motivate your answer.

Please turn over for the test of Linear Optimization