WORK OUT ON A PAPER SEPARATE FROM LINEAR OPTIMIZATION

Module 2, test 2 Analysis, 201300057

23-1-2017,

Motivate all your answers.

1. Decide which of the following statements are true and which are false. Prove the true ones and provide counterexamples for the false ones.

   (a) (3pt) Let f and g be real functions on an open interval I with $a \in I$. If $\lim_{x\to a} f(x)$ does not exist and $f(x) \leq g(x)$ for all $x \in I$, then $\lim_{x\to a} g(x)$ doesnt exist either.

   (b) (3pt) The function f on $(0,1)$ defined by $f(x) = x \log \frac{1}{x}$ is uniformly continuous.

2. (a) (2 pt) Formulate the Inverse Function Theorem.

   (b) (4 pt) Suppose that I is a nondegenerate interval. Let $f : I \to \mathbb{R}$ be a differentiable function with $f'(x) \neq 0$ for all $x \in I$.
   Prove that f^{-1} exist on $f(I)$ and is differentiable on $f(I)$.

3. (a) (2 pt) Give the definition of a Riemann integrable function.

   (b) (4 pt) Suppose that $a, b \in \mathbb{R}$ with $a < b$. Proof the following statement:
   If f is continuous on the interval $[a, b]$, then f is integrable on $[a, b]$.

   Hint: f is continuous on $[a, b]$, so f is uniformly continuous.

Please turn over for the test of Linear Optimization