WORK OUT ON A PAPER SEPARATE FROM ANALYSIS

Test 1 Linear Optimization (course code 201300057)

Give motivations for all your answers.

1. (8 pt) We are interested in the amount of water that can flow through a network of tubes. Consider a network that consists of a set of control points $P = \{1, \ldots, N\}$. If there exists a tube such that water can flow from control point $i \in P$ to control point $j \in P$, we say j is a neighbour of i. We denote the set of all neighbours of $i \in P$ by N_i. Each tube from $i \in P$ to $j \in N_i$ has a capacity c_{ij} that denotes the amount of water that can flow though that tube.
   An example is shown in figure 1. Note that in this example at most two units of water can flow from control point 1 to N; one unit flows from control point 1 through 2 and 4 to 5, and one unit flows from 1 through 3 and 4 to 5.
   Formulate a linear program that finds the largest amount of water that can flow from node 1 to node N in a general network (do **not** just formulate an LP for the example in figure 1).

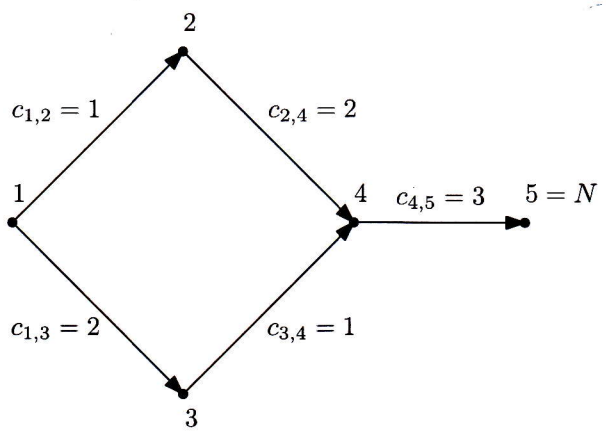

Figure 1: Dots represent control points. An arrow from control point i to j represents a tube in which water flows from i to j.

2. (10 pt) For each of the following statements, determine whether it is true or false. Provide either a short argument of at most two lines, or a (counter)example.

   a) The program $\{\min x, \text{s.t.} |x-1| \leq 3\}$, where $x \in \mathbb{R}$, can be formulated as a linear program.

   b) The union of a finite number of convex sets is a convex set.

   c) Every basic solution is an extreme point.

   d) If two distinct bases of a standard form polyhedron with independent rows lead to the same basic solution, then that solution is degenerate.

   e) If $\mathbf{x}$ is a basic feasible solution that is associated with a basis matrix with reduced costs $\bar{\mathbf{c}} \geq 0$, then $\mathbf{x}$ is optimal.

Please turn over for the test of Analysis