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Dynamica

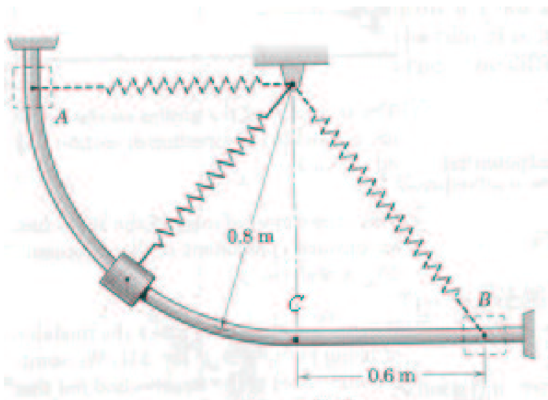
Exam II

This is an open book exam. You can use any book, any lecture note, and any notes from the werkcollege. You can use a pocket calculator, but no laptops. Also, you may not consult anybody else, apart from the docenten (if, for example, you have problems with the language).
The tentamen consists of 7 problems. As you have 3.5 hours (13:30–17:00), this means 30 minutes per problem, so don't panic.
The total number of points in this tentamen is **100**. You do *not* need to solve all problems correctly to get a cijfer 10.

1) **Slider (15 points)**

A 3-kg slider and attached massless spring are released from rest at A; it slides with negligible friction in the vertical plane along the rod. The attached spring has a stiffness of 200 N/m and an unstretched length of 0.40 m.

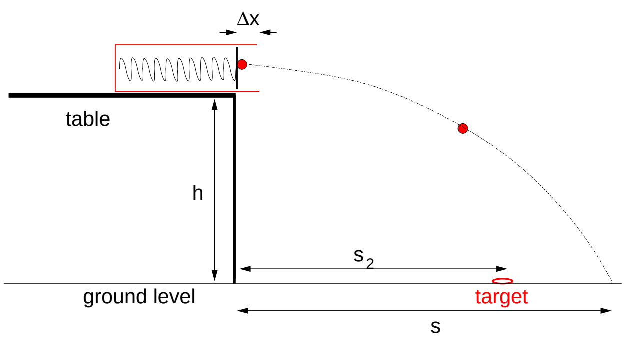
(a) Does the spring force do work in the circular part of the rod? If yes, calculate the work done by this force in this part of the rod. If your answer is no, state why not.
(b) Determine the velocity of the slider as it passes position C.
(c) Calculate the velocity of the slider as it reaches B.
(d) How far does the slider travel to the right of point B, before it reverses direction?



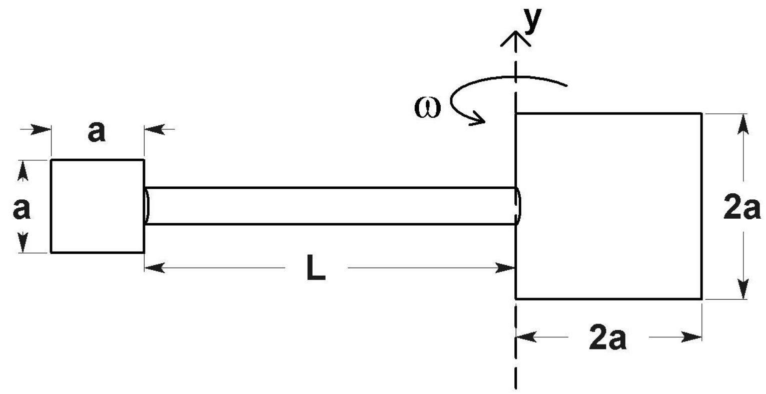
2) **Pea-gun (13 points)**

A children's toy consists of a spring and a massless plate which shoots dried peas. The pea-gun is set upon a horizontal table. In a first try, the spring is compressed by $\Delta x_1 = 2.0 cm$, and the shot travels $s_1 = 2.50$ m before hitting the ground.

(a) If you want to hit a target on the ground located $s_2 = 2.00$ m from the edge of the table, how much must you compress the spring?
(b) Alternatively, you could change the height of the table, leaving $\Delta x = \Delta x_1$ the same. Assuming the initial height is $h_1 = 0.8$ m, what is the new height in order to hit the target?
(c) If the force needed to compress the spring by Δx_1 is $F_1 = 4.0$ N, what is the mass of one pea?



3) **Wind vane (16 points)**

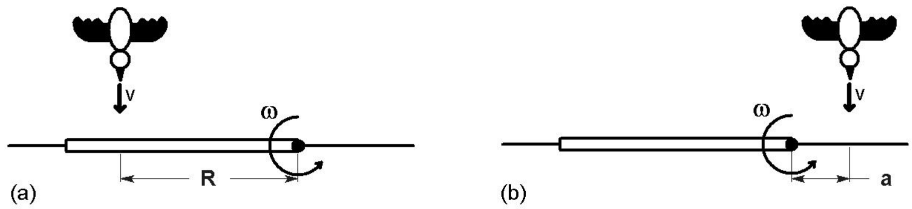


A wind vane consists of two vertical, light-metal, square slabs of equal density and equal thickness, and a thin rod connecting them. The squares have edge length a and $2a$, respectively, and the thin rod has length L. The thickness of the squares and the diameter of the rod are negligibly small. The whole wind vane will rotate around the y-axis as the wind blows against it.

a) If the large ($2a \times 2a$) slab has a mass of m_s and the thin rod has a mass of m_r, calculate the rotational inertia (I) of the wind vane rotating around the y-axis as a function of m_s, m_r, a, and L.

b) The wind vane is balanced such that its center-of-mass lies on the y-axis. With this information, calculate the mass of the rod m_r, if $m_s = 2.4\,\text{kg}$, $a = 0.40\,\text{m}$, $L = 0.80\,\text{m}$.

If you did not solve (a), assume that $I = 1.5\,\mathbf{kg\,m^2}$ from now on!
Due to a breeze, the wind vane starts to rotate with constant angular velocity $\omega = 1.0$ rad/s.



c) A small bird of mass $m_b = 0.40$ kg now lands on the thin rod a distance $R = 0.60\,\text{m}$ from the y-axis as depicted in the part (a) of the figure above. If its landing speed is $V = 1.2\,\text{m/s}$, what is the kinetic energy increase of the wind vane?

As the bird takes off again, the wind vane returns to the constant angular velocity $\omega = 1.0$ rad/s.

d) A second bird tries to land on the wind vane. This bird is less fortunate as it doesn't land, but **collides elastically** with the center of the wind vane's large square (see figure, part (b)). Calculate the decrease in kinetic energy of the wind vane, if the collision speed V of the bird is again 1.2 m/s.

No birds were harmed in the making of this exercise.

4) **Dumbbell (13 points)**

A dumbbell with mass M and uniform composition consists of a cylinder with radius R_1 and length l, with at the ends two cylinders with radius R_2 and height $\frac{l}{2}$. The dumbbell rolls from an inclined slope without slipping, with the larger cylinders hanging over the sides of the slope (see figure).

a) Calculate the rotational intertia of the dumbbell about its major axis.

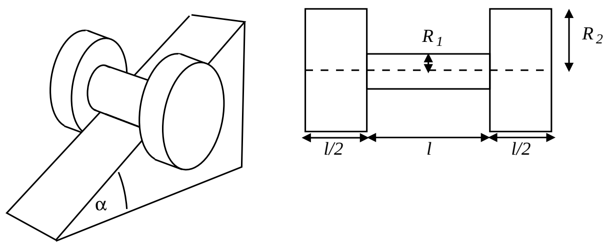
b) Show that the acceleration of the dumbbell is given by:

$$a_{cm} = 2g\sin(\alpha)\frac{R_1^4 + R_1^2R_2^2}{3R_1^4 + 2R_1^2R_2^2 + R_2^4}$$

From here on, assume $R_1 = 2.0$ cm, $R_2 = 7.5$ cm, and $l = 15$ cm. The angle of the slope is $\alpha = 30\,\text{deg}$.

c) What is the minimal value of the coefficient of static friction μ_s to prevent the dumbbell from slipping?

d) How long does it take for the dumbbell to reach the end of an $L = 50$cm slope?



5) **Supernova remnant (16 points)**

A supernova is a stellar explosion that often leaves behind only a circular ring of matter. Suppose this ring is compact, so that all the mass m is concentrated at a radius r (figure). A small mass m_0 is introduced at a distance y from the center of the ring along its axis.

(a) Show that the net gravitational force exerted by the ring onto the mass is

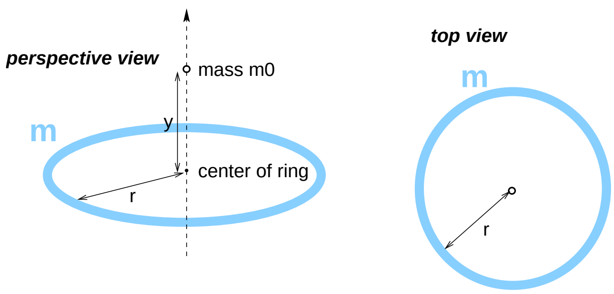
$$F = \frac{Gmm_0y}{(y^2 + r^2)^{3/2}}\,!$$

Hint: every length element of the ring acts like a point mass.

(b) Derive an expression for the velocity of the particle at the center of the ring, if it falls from rest from a distance $y = y_0$!

(c) Simplify this expression for $y_0 \ll r$ (Taylor expansion!), and show that the velocity at the center is then approximately $v(0) = \sqrt{Gm\frac{y_0}{r^{3/2}}}$!

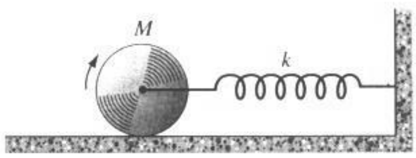
(d) Again, let $y_0 \ll r$. Show that the falling mass m_0 will undergo an approximately harmonic oscillation about the center of the ring, and compute the frequency of this oscillation!



6) **Cylinder oscillation (14 points)**

A solid cylinder is attached to a horizontal massless spring so that it can roll without slipping along a horizontal surface, as shown in the figure. The force constant of the spring is $k = 2.5\,\mathrm{N/cm}$. If the system is released from rest at a position in which the spring is stretched by 30.0 cm, find:
a) the translational kinetic energy of the cylinder as it passes through the equilibrium position,
b) the rotational kinetic energy of the cylinder as it passes through the equilibrium position.
c) Show that under these conditions the center of mass of the cylinder executes simple harmonic motion with a period

$$T = 2\pi\sqrt{3M/2k}\,,$$



where M is the mass of the cylinder.

7) **Rope (13 points)**

A piece of rope of mass M and total length L is put on a table, with a piece of $L/3$ dangling over the edge of the table (figure), so that the lower end is initially at $z(0) = -L/3$ as counted from the table top. Initially the rope is held in place by a person, and thus at rest. When released, the piece of rope will slide off the table. While sliding, the rope is affected by a friction force, with a magnitude equal to $\mu|\vec{N}|$, with $\vec{N}$ the normal force between table and rope.
(a) Using a force balance, write down the differential equation of motion for the position of the lower end of the rope, $z(t)$, which is valid up to the point where the rope has left the table completely.
(b) Show that the differential equation in (a), with the initial conditions given, is solved by a function of the type

$$z(t) = Ae^{\omega t} + Be^{-\omega t} + C\,,$$

and determine the constants A, B, C and ω for the solution.