1 (i) $0 \leq P(E) \leq 1$

(ii) $P(S) = 1$

3 (iii) $P(\cup E_n) = \sum_n P(E_n)$ if $E_n E_m = \emptyset$ $(n \neq m$

Ross p. 4

2 $+ =$ "positief", $D =$ "doping"

1 geg: $P(+|D) = 0{,}85$, $P(+|D^c) = 0{,}05$, $P(D) = 0{,}1($

1 dus $P(+) = P(+|D)P(D) + P(+|D^c)P(D^c) = 0{,}13$

2 $$P(D|+) = \frac{P(+|D)P(D)}{P(+)} = 0{,}654 = \frac{17}{26}$$

3 a $$P(X_1 = i) = \sum_{j=0}^{\infty} P(X_1 = i, X_2 = j)$$

2 $$= \sum_{j=0}^{\infty} e^{-2} \frac{1}{i!} \frac{1}{j!} = e^{-1} \frac{1}{i!} \quad i = 0, 1, \ldots$$

b $$P(X_2 = j) = e^{-1} \frac{1}{j!}, \quad j = 0, 1, \ldots$$

2 $P(X_1 = i, X_2 = j) = P(X_1 = i) P(X_2 = j)$ dus o.o.

c $$P(X_1 + X_2 = k) = \sum_{i=0}^{\infty} P(X_1 = i, X_2 = k - i)$$

2 $$= \sum_{i=0}^{k} e^{-2} \frac{1}{i!} \frac{1}{(k-i)!} = \frac{1}{k!} e^{-2} \sum_{i=0}^{k} \binom{k}{i} = \frac{2^k}{k!} e^{-k}$$

$k = 0, 1, \ldots$

dus Poisson met $\mu = 2$

4 a. $f_X(x) = \begin{cases} \frac{1}{\pi} & 0 < x < \pi \\ 0 & \text{elders} \end{cases}$

2 $\varphi_X(t) = E[e^{tX}] = \int_0^{\pi} \frac{1}{\pi} e^{tx}\, dx$

$= \frac{1}{\pi t}(e^{\pi t} - 1)$

b $E[\sin X] = \int_0^{\pi} \sin x \frac{1}{\pi}\, dx$

2 $= -\frac{1}{\pi} \cos x \Big|_0^{\pi} = \frac{2}{\pi}$

c $F_Y(x) = P(Y \leq x) = P(X^2 \leq x)$

$= \begin{cases} 0 & x \leq 0 \\ \frac{1}{\pi}\sqrt{x} & 0 < x < \pi^2 \\ 1 & x \geq \pi^2 \end{cases}$

3

dus

$f_Y(x) = \begin{cases} \frac{1}{2\pi\sqrt{x}} & 0 < x < \pi^2 \\ 0 & \text{elders} \end{cases}$

5 a $EX^2 = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx = \int_0^1 x^3\, dx + \int_1^2 x^2(2-x)\, dx$

$= \frac{1}{4} + \frac{11}{12} = \frac{7}{6}$ $\qquad EX = 1$

2

$\text{var}\, X = EX^2 - (EX)^2 = \frac{7}{6} - 1 = \frac{1}{6}$

b $ES_n = n\, EX_1 = n$

2

$\text{var}\, S_n = n\, \text{var} X_1 = \frac{1}{6} n$

c $\quad \operatorname{cov}(S_n, S_{n+1}) = \operatorname{cov}(S_n, S_n + X_{n+1})$

2 $\quad = \operatorname{cov}(S_n, S_n) + \operatorname{cov}(S_n, X_{n+1})$

$= \operatorname{var} S_n = \frac{1}{6} n$

1 d $\quad$ C.L.S. : $\quad \lim\limits_{n\to\infty} P\left(\frac{S_n - ES_n}{\sqrt{\operatorname{var} S_n}} \leq x \right) = \Phi(x)$

$\Leftrightarrow \quad \lim\limits_{n\to\infty} P\left(\frac{S_n - n}{\sqrt{n/6}} \leq x \right) = \Phi(x)$

$\Leftrightarrow \quad \lim\limits_{n\to\infty} P\left(\frac{S_n - n}{\sqrt{n}} \leq \frac{x}{\sqrt{6}} \right) = \Phi(x)$

dus

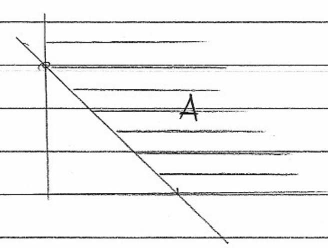
1 $\quad \lim\limits_{n\to\infty} P\left(\frac{S_n - n}{\sqrt{n}} \leq 1 \right) = \Phi(\sqrt{6})$

6 a $\quad P(Z > z) = P(X > z, Y > z) = e^{-\frac{z}{5}} \cdot e^{-\frac{z}{5}} = e^{-\frac{2}{5}z}$

2 $\quad$ dus $P(Z \leq z) = 1 - e^{-\frac{2}{5}z}, \quad z \geq 0$

$E(Z) = 1/\tfrac{2}{5} = 2{,}5$

b



1 $\quad P(X + Y > 1) = \iint\limits_A f_{X,Y}(x,y)\,dy\,dx$

$$= 1 - \int_0^1 \int_0^{1-x} \frac{1}{5} e^{-\frac{1}{5}x} \, \frac{1}{5} e^{-\frac{1}{5}y} \, dy \, dx$$

$$= 1 - \int_0^1 \frac{1}{5} e^{-\frac{1}{5}x} \left(1 - e^{-\frac{1}{5}(1-x)}\right) dx$$

2

$$= 1 - \int_0^1 \frac{1}{5} e^{-\frac{1}{5}x} \, dx + \int_0^1 \frac{1}{5} e^{-\frac{1}{5}} \, dx$$

$$= 1 - \left(1 - e^{-\frac{1}{5}}\right) + \frac{1}{5} e^{-\frac{1}{5}} = \frac{6}{5} e^{-\frac{1}{5}} = 0{,}982$$

1 c $E[X \mid X > 10] = 10 + EX = 15$

2 $E[X \mid X > Y] = E[E[X \mid X > Y, Y]]$

$$= E[Y + EX] = E[Y] + E[X] = 10$$