1\. a. $Z =$ "zwanger", $T =$ "test positief"

$P(\bar{Z}|T) = 0{,}6 \quad P(\bar{T}|\bar{Z}) = 0{,}9 \quad P(T|Z) = 1$

b Zij $p = P(Z)$, dan (Bayes)

$$0{,}6 = P(\bar{Z}|T) = \frac{P(T|\bar{Z})P(\bar{Z})}{P(T|Z)P(Z) + P(T|\bar{Z})P(\bar{Z})}$$

$$= \frac{(1-0{,}9)(1-p)}{p+(1-0{,}9)(1-p)} \Rightarrow p = \frac{1}{16} = 0{,}0625$$

2\. Noem het aantal keren dat het door jou gekozen ogenaantal voorkomt N en de winst W; N is $B(3;\frac{1}{6})$-verdeeld en $W=-1$ als $N=0$ en $W=N$ voor de overige waarden van N. Dus:

$$EW = -1\times(\tfrac{5}{6})^3 + 1\times3\times\tfrac{1}{6}\times(\tfrac{5}{6})^2 + 2\times3\times(\tfrac{1}{6})^2\times\tfrac{5}{6} + 3\times(\tfrac{1}{6})^3 = -\tfrac{17}{216} =$$

$$= -0{,}0787.$$

3 a. $F_Y(y) = P(Y \leq y) = P(\sqrt{X} \leq y)$

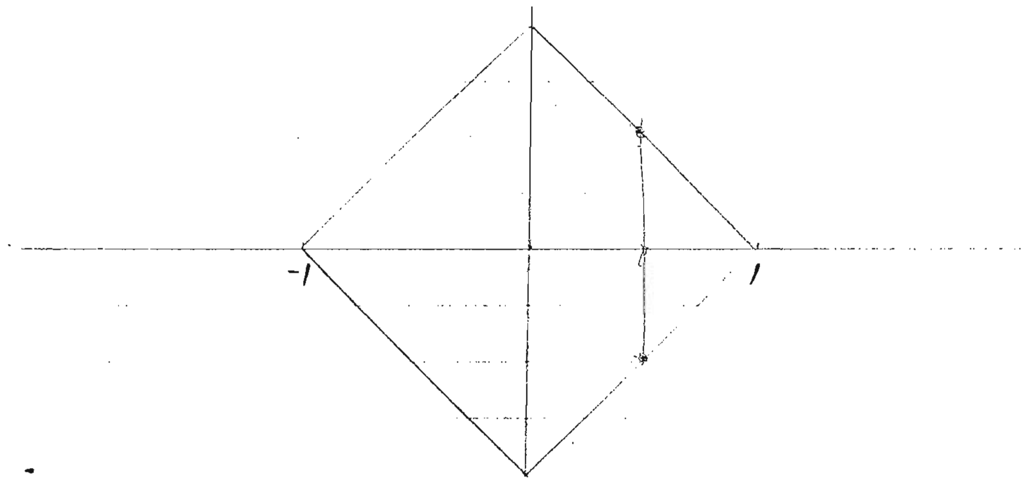
$= P(X \leq y^2) = 1 - e^{-\frac{1}{2}y^2}, \quad y \geq 0$

$f_Y(y) = \frac{d}{dy} F_Y(y) = y e^{-\frac{1}{2}y^2}, \quad y \geq 0$

b. $EY = \int_0^\infty y^2 e^{-\frac{1}{2}y^2} dy = \frac{1}{2}\sqrt{2\pi}\int_{-\infty}^{\infty} y^2 \varphi(y)\, dy = \sqrt{\frac{\pi}{2}}$

want $\operatorname{var} Z = EZ^2 = 1$ als $Z \sim N(0,1)$

4




a.

$$1 = \iint f_{X,Y}(x,y)\,dx\,dy = 2c \quad \Longrightarrow \quad c = \tfrac{1}{2}$$

b.

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

$$= \begin{cases} \displaystyle\int_{|x|-1}^{1-|x|} \tfrac{1}{2}\,dy = 1-|x| & -1<x<1 \\ 0 & \text{elders} \end{cases}$$

c.

$$f_Y(y) = \begin{cases} 1-|y| & -1<y<1 \\ 0 & \text{elders} \end{cases}$$

$$EX = EY = \int_{-1}^{1} y(1-|y|)\,dy = 0$$

$$EXY = \iint_{\Diamond} \tfrac{1}{2}\,xy\,dx\,dy = \tfrac{1}{2}\int_{-1}^{1} y\left\{\int_{-1+|y|}^{1-|y|} x\,dx\right\}dy$$

$$= \tfrac{1}{2}\int_{-1}^{1} 0\;dy = 0$$

Dus $\operatorname{cov}(X,Y) = EXY - EX\,EY = 0$

ongecorreleerd !

d. $f_{X,Y}(x,y) \neq f_X(x) f_Y(y)$ dus niet o.o.

e. $f_{X|Y}(x|y)$ is gedefinieerd voor

$f_Y(y) > 0$, d.w.z. $-1 < y < 1$. Dan

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \begin{cases} \frac{1}{2(1-|y|)} & |x|+|y|<1 \\ 0 & |x|+|y| \geq 1 \end{cases}$$

f.
$$E(X^2 | Y = \tfrac{1}{2}) = \int_{-\infty}^{\infty} x^2 f_{X|Y}(x|\tfrac{1}{2})\, dx$$

$$= \int_{-1/2}^{1/2} x^2\, dx = \frac{1}{3} \cdot \frac{2}{8} = \frac{1}{12}$$

5 a.
$$f_{S_2}(z) = \int_{-\infty}^{\infty} f_{X_1}(x) f_{X_2}(z-x)\, dx$$

$$= \int_{\max(-1, z-1)}^{\min(1, z+1)} \tfrac{1}{4}\, dx \qquad -2 < z < 2$$

$$= \tfrac{1}{4}(2 - |z|) \qquad -2 < z < 2$$

b. $EX_i = 0$, $\operatorname{var} X_i = \frac{1}{12}(b-a)^2 = \frac{1}{3}$

$ES_n = n\, EX_1 = 0 \qquad \operatorname{var} S_n = n \operatorname{var} X_1 = \frac{1}{3} n$

c. $\varphi_{X_1}(t) = E e^{tX_1} = \int_{-1}^{1} \frac{1}{2} e^{tx}\, dx = \frac{1}{2t}(e^t - e^{-t})$

$\varphi_{S_n}(t) = (\varphi_{X_1}(t))^n = (\frac{1}{2t})^n (e^t - e^{-t})^n$

d. $P(\frac{1}{100}|S_{100}| < 0{,}1) = P(|S_{100}| < 10)$

$= P\left(-\frac{10}{\sqrt{100/3}} < \frac{S_{100}}{\sqrt{100/3}} < \frac{10}{\sqrt{100/3}}\right)$

$= P\left(-\sqrt{3} < \frac{S_{100}}{\sqrt{100/3}} < \sqrt{3}\right) \approx 2\Phi(\sqrt{3}) - 1$

6 a $U = \min(X_1, X_2)$

$P(U > u) = P(X_1 > u, X_2 > u) = e^{-2\lambda u}, \quad u \geq 0$

$\therefore F_U(u) = P(U \leq u) = 1 - e^{-2\lambda u}, \quad u \geq 0$

b $V = \max(X_1, X_2)$

$F_V(v) = P(V \leq v) = P(X_1 \leq v, X_2 \leq v) = (1 - e^{-\lambda v})^2, \quad v \geq 0$

$U + V = X_1 + X_2 \Rightarrow EV = EX_1 + EX_2 - EU = \frac{3}{2\lambda}$

c $P(T_1 > T_2 + t) = \int_0^\infty P(T_1 > T_2 + t \mid T_2 = u)\lambda e^{-\lambda u}\,du$

$= \int_0^\infty e^{-\lambda(t+u)} \lambda e^{-\lambda u}\,du = \frac{1}{2} e^{-\lambda t}, \quad t \geq 0$

$\therefore P(W > t \mid T_1 > T_2) = P(T_1 > T_2 + t \mid T_1 > T_2 + 0) = e^{-\lambda t}, \quad t \geq 0$

d. vanwege symmetrie

$P(W > t) = P(W > t \mid T_1 > T_2)P(T_1 > T_2) + P(W > t \mid T_2 > T_1)P(T_2 > T_1)$

$= 2P(W > t \mid T_1 > T_2)P(T_1 > T_2) = e^{-\lambda t}, \quad t \geq 0$

Dus $W \sim Exp(\lambda)$