1 $P(G) = P(O) = \frac{1}{2}$

a. $P(K|G) = \frac{1}{2} \quad P(K|O) = 1$

2 $P(K) = P(K|G)P(G) + P(K|O)P(O) = \frac{3}{4}$

$P(G|K) = P(K|G)P(G)/P(K) = \frac{1}{3}$

b $P(KK|G) = \frac{1}{4} \quad P(KK|O) = 1$

$P(KK) = P(KK|G)P(G) + P(KK|O)P(O) = \frac{5}{8}$

2 $P(G|KK) = P(KK|G)P(G)/P(KK) = \frac{1}{5}$

1 c $P(G|KKM) = 1$

2 a

1 $c \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{2^i}{i!} \frac{3^j}{j!} = ce^6 = 1 \Rightarrow c = e^{-6}$

b $P(X=i) = \sum_{j=0}^{\infty} P(X=i, Y=j) = e^{-2} \frac{2^i}{i!}, \quad i = 0, 1, \ldots$

2 $EX = 2$

c $\varphi_X(t) = Ee^{tX} = \sum_{i=0}^{\infty} e^{ti} e^{-2} \frac{2^i}{i!} = e^{2(e^t - 1)}$

$\varphi_X''(t) = 4e^{2t} e^{2(e^t-1)} + 2e^t e^{2(e^t-1)}$

2 $\Rightarrow EX^2 = \varphi_X''(0) = 6$

$\Rightarrow \operatorname{var} X = EX^2 - (EX)^2 = 2$ (of direct) d $P(X=i, Y=j) = P(X=i)P(Y=j)$ dus o.o.

2 $\Rightarrow P(X=i \mid Y=0) = P(X=i) = e^{-2}\frac{2^i}{i!} \quad i=0,1,\ldots$

e $\operatorname{cov}(X, X+Y) = \operatorname{cov}(X,X) + \operatorname{cov}(X,Y)$

2 $= \operatorname{var} X = 2$

1 3 a $\frac{1}{2}$

1 b $P(W_y = 0) = P(T_y \leq T_K) = \frac{1}{2}$

c $0 \leq x \leq 1$:

$$P(W_y \leq x) = P(W_y = 0) + P(W_y \leq x, W_y > 0)$$

$$= \frac{1}{2} + P(0 < T_K - T_y \leq x)$$

$$= \frac{1}{2} + \int_0^1 P(T_y < T_K \leq T_y + x \mid T_y = u)\,du$$

2 $$= \frac{1}{2} + \int_0^1 P(u < T_K \leq u + x)\,du$$

$$= \frac{1}{2} + \int_0^{1-x} x\,du + \int_{1-x}^1 (1-u)\,du$$

$$= \begin{cases} \frac{1}{2} + x - \frac{1}{2}x^2 & 0 \leq x < 1 \\ 0 & x < 0 \\ 1 & x \geq 1 \end{cases}$$

d $P(W_j \leq x \mid W_j > 0) = 2P(0 < W_j \leq x)$

$$= \begin{cases} 2x - x^2 & 0 \leq x < 1 \\ 0 & x < 0 \\ 1 & x \geq 1 \end{cases}$$

2

$\therefore f_{W_j|W_j>0}(x) = 2(1-x), \quad 0 \leq x \leq 1$

$\therefore E[W_j] = \frac{1}{2} E[W_j \mid W_j > 0]$

$$= \frac{1}{2}\int_0^1 x f_{W_j|W_j>0}(x)\,dx = \frac{1}{6}$$

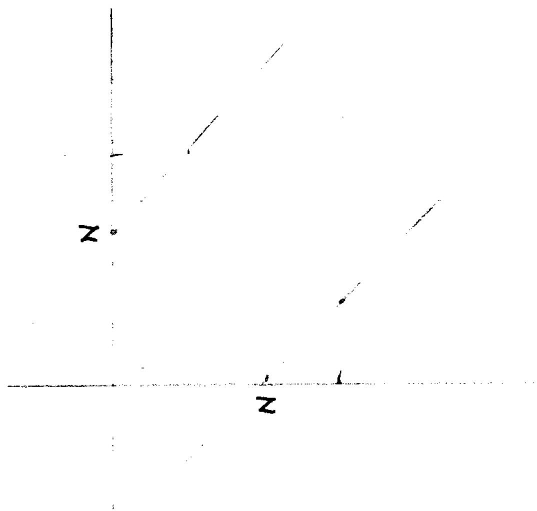
e $T_j \geq T_k \Rightarrow W_j = 0 \quad W_k = T_j - T_k$

1 $T_j < T_k \Rightarrow W_k = 0 \quad W_j = T_k - T_j$

$\therefore W_j + W_k = |T_j - T_k|$

f

$P(W_j + W_k \leq z) =$
$P(|T_j - T_k| \leq z) =$



$1 - (1-z)^2, \quad 0 \leq z \leq 1$

2

$\therefore$

$$f_{W_j+W_k}(z) = \begin{cases} 2(1-z) & 0 \leq z \leq 1 \\ 0 & \text{elders} \end{cases}$$

4 a. $EX_1 = \int_0^1 \frac{3}{2} x\sqrt{x}\, dx = \frac{3}{5}$

$EX_1^2 = \int_0^1 \frac{3}{2} x^2\sqrt{x}\, dx = \frac{3}{7}$

$\operatorname{var} X_1 = EX_1^2 - (EX_1)^2 = \frac{12}{175}$

$\therefore \quad ES_n = \frac{3}{5} n \qquad \operatorname{var} S_n = \frac{12}{175} n$

b $\quad E S_{525} = 525 \cdot \frac{3}{5} = 315$

$\operatorname{var} S_{525} = 525 \cdot \frac{12}{175} = 36$

C.L.S. $\quad P\left(\frac{S_{525} - 315}{6} \leq x \right) \approx \Phi(x)$

$\therefore \quad P(|S_{525} - 315| < 12)$

$= P\left(-2 < \frac{S_{525} - 315}{6} < 2 \right)$

$\approx \Phi(2) - \Phi(-2) = 2\Phi(2) - 1$

c $\quad$ Chebysev: $P(|S_{525} - ES_{525}| \geq k) \leq \frac{\operatorname{var} S_{525}}{k^2}$

$\therefore$

$P(|S_{525} - 315| \geq 12) \leq \frac{36}{144} = \frac{1}{4}$

$\therefore \quad P(|S_{525} - 315| < 12) \geq \frac{3}{4}.$

2 5 a $E[X \mid X > c] = c + EX = c + \frac{1}{\lambda}$

b $E[X \mid X > Y] = E[\min(X,Y) + X - Y \mid X > Y]$

2 $= E[\min(X,Y) \mid X > Y] + E[X - Y \mid X > Y]$

$= E[\min(X,Y)] + \frac{1}{\lambda} = \frac{1}{\lambda+\mu} + \frac{1}{\lambda}$

c $f_{X+Y}(z) = \int_0^\infty f_X(x) f_Y(z-x)\, dx$

$= \int_0^z \lambda e^{-\lambda x} \mu e^{-\mu(z-x)}\, dx$

3 $= \lambda\mu e^{-\mu z} \int_0^z e^{-(\lambda-\mu)x}\, dx$

$= \frac{\lambda\mu}{\lambda-\mu} e^{-\mu z}\left(1 - e^{-(\lambda-\mu)z}\right)$

$= \frac{\lambda}{\lambda-\mu} \mu e^{-\mu z} + \frac{\mu}{\mu-\lambda} \lambda e^{-\lambda z}, \quad z \geq 0$