Uitwerking tentamen Calculus II, TW. (20-03-98)

1. $f(x,y)=\ln(xy)+x^2+x^3y \quad (1,1) \Rightarrow 2$

$f_x = \frac{1}{x}+2x+3x^2y \quad \Rightarrow 6$

$f_y = \frac{1}{y}+x^3 \quad \Rightarrow 2$

$f_{xx} = -\frac{1}{x^2}+2+6xy \quad \Rightarrow 7$

$f_{xy} = 3x^2 \quad \Rightarrow 3$

$f_{yy} = -\frac{1}{y^2} \quad \Rightarrow -1$

$$f(1+h_1,1+h_2)=2+6h_1+2h_2+\frac{7}{2}h_1^2+3h_1h_2-\frac{1}{2}h_2^2+o(\|(h_1,h_2)\|^2)$$

2 a 1e $f(1,0,0,0)=(0,0)$
2e f van klasse C^1 $\Rightarrow$ I.F.st.
3e $\frac{\partial f}{\partial v,\omega} = \begin{vmatrix} 1 & 2\omega \\ 0 & x \end{vmatrix}_{(1,0,0,0)} = x\Big|_{(1,0,0,0)} = 1 \neq 0$

$y^2+v+\omega^2=0 \qquad 0+v_x+2\omega\omega_x=0$
$x\omega=0 \qquad \omega+x\omega_x=0 \quad \Big|_{(1,0,0,0)} \Rightarrow \omega_x=0 \Rightarrow v_x=0$

$\left.\begin{array}{l} 2y+v_y+2\omega\omega_y=0 \\ x\omega_y=0 \end{array}\right\}(1,0,0,0) \quad \omega_y=0 \Rightarrow v_y=0$

c niet locaal C^1 inverteerbaar t.g.v. $\left|\begin{bmatrix} v_x & v_y \\ \omega_x & \omega_y \end{bmatrix}\right|=0$

3 a inwendige van A : $\mathring{A}$

$$\left.\begin{array}{ll} f_{x_1}=0 & 2x_1 - x_2 = 0 \\ f_{x_2}=0 & 2x_2 - x_1 = 0 \end{array}\right\} \Rightarrow (0,0) \Rightarrow f(0,0)=0$$

$$f(x_1,x_2) = (x_1^2 + x_2^2 - x_1x_2) = \frac{1}{2}(x_1-x_2)^2 + \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$$

$$f_{x_1x_1}=2 \qquad f_{x_2x_2}=2$$

$$f_{x_1x_2}=-1 \qquad \Rightarrow H_f = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 5 > 0$$

$\Rightarrow$ locaal minimum

rand van A

grad $f = \lambda$ grad g

$g = 0$

$$2x_1 - x_2 = 2\lambda x_1 \qquad x_2(2x_1-x_2) = x_1(2x_2-x_1)$$

$$2x_2 - x_1 = 2\lambda x_2$$

$$x_1^2 + x_2^2 = 1 \qquad -x_2^2 = -x_1^2 \qquad x_1 = \pm x_2$$

$$\Rightarrow \left(\tfrac{1}{2}\sqrt{2}, -\tfrac{1}{2}\sqrt{2}\right) \quad \left(\tfrac{1}{2}\sqrt{2}, +\tfrac{1}{2}\sqrt{2}\right) \quad \left(-\tfrac{1}{2}\sqrt{2}, \tfrac{1}{2}\sqrt{2}\right). \quad \left(-\tfrac{1}{2}\sqrt{2}, -\tfrac{1}{2}\sqrt{2}\right)$$

$$\downarrow$$

$$\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2} \qquad \tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{2} \qquad \tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2} \qquad \tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{2}$$

$$\Downarrow$$

$$3/2 \qquad 1/2 \qquad 3/2 \qquad 1/2$$

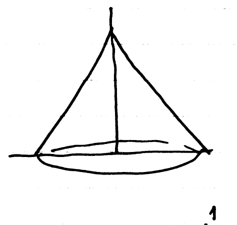
b $\Rightarrow$ Weierstrass globaal max : $\left(\tfrac{1}{2}\sqrt{2}, -\tfrac{1}{2}\sqrt{2}\right)\left(-\tfrac{1}{2}\sqrt{2}, \tfrac{1}{2}\sqrt{2}\right) \Rightarrow \frac{3}{2}$

" min : $(0,0)$ $\Rightarrow 0$

4 $x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$



kegel

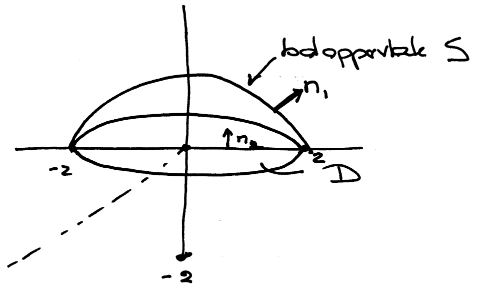
Jacobiaan : r

$$\int_0^{2\pi}\left[\int_0^1\left[\int_0^{1-r} zr\,dz\right]dr\right]d\varphi = 2\pi\int_0^1 \frac{1}{2}z^2 r\Big|_0^{1-r} dr =$$

2e gedeelte zie na 5

$$= \pi \int_0^1 r(1+r^2-2r)\,dr = \pi \int_0^1 r^3-2r^2+r\,dr = \pi\left(\frac{1}{4}-\frac{2}{3}+\frac{1}{2}\right) = \frac{\pi}{12}$$

5 a -
b - dictaat



c $\displaystyle\iint_S \text{rot}\,\underline{v}\cdot\underline{n}_1\,dA = \oint_{\substack{\partial S\\ \partial D}} \underline{v}\,d\underline{r}$

∂S $x = \cos\varphi$, $y = \sin\varphi$, $z = 0$

$x_\varphi = -\sin\varphi$, $y_\varphi = \cos\varphi$, $z_\varphi = 0$

$\underline{v} \Rightarrow (\cos^2\varphi, \cos^3\varphi, -1)$

$$\oint_{\partial S} \underline{v}\,d\underline{r} = \int_0^{2\pi} \cos^2\varphi\cdot -\sin\varphi + \cos^3\varphi\cdot\cos\varphi\,d\varphi$$

$$= \frac{1}{3}\cos^3\varphi\Big|_0^{2\pi} + \int_0^{2\pi}\cos^4\varphi\,d\varphi = \int_0^{2\pi}\left(\frac{1}{2}+\frac{1}{2}\cos 2\varphi\right)^2 d\varphi$$

$$= \int_0^{2\pi} \frac{1}{4} + \frac{1}{4}\cos^2 2\varphi + \cancel{\frac{1}{2}\cos 2\varphi}\,d\varphi = \frac{1}{4}\cdot 2\pi + \frac{1}{4}\cdot\pi = \frac{3\pi}{4}$$

4b 2e gedeelte

$z = r\cos\vartheta$ $\qquad \vartheta: \frac{\pi}{2} \to \pi$

$x = r\sin\vartheta\cos\varphi$ $\qquad \varphi: 0 \to 2\pi$ $\qquad J = r^2\sin\vartheta$

$y = r\sin\vartheta\sin\varphi$ $\qquad r: 0 \to 1$

$$= \int_{\frac{\pi}{2}}^{\pi}\int_0^{2\pi}\int_0^1 r\cos\vartheta\cdot r^2\sin\vartheta\,dr\,d\varphi\,d\vartheta = \frac{1}{4}\cdot 2\pi\int_{\frac{\pi}{2}}^{\pi}\sin\vartheta\cos\vartheta\,d\vartheta$$

$$= \frac{1}{4}\pi\int_{\frac{\pi}{2}}^{\pi}\sin 2\vartheta = \frac{1}{4}\pi\cdot -\frac{1}{2}\cos 2\vartheta\Big|_{\frac{\pi}{2}}^{\pi} = -\frac{1}{8}\pi\,(1-(-1)) = -\frac{1}{4}\pi$$