I

Uitwerking Calculus II d.d 24 augustus 1999
(Behoudens rekenfouten!!)

1 Impliciete functiestelling

(α) f klasse C^2

(β) $\left|\left[\frac{\partial f}{\partial y}\right]\right| = \frac{\partial(f_1, f_2)}{\partial(x_1, x_2)} = \left|\begin{bmatrix} \sin x_2 & x_1\cos x_2 + \sin x_3 \\ 2x_1 + x_3 & 0 \end{bmatrix}\right|_{(1,0,0)} = \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -2$

(γ) $f(1,0,0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \square$

b

$\varphi_1 \sin\varphi_2 + \varphi_2 \sin x_3 = 0$

$\varphi_1^2 + \varphi_1 x_3 - 1 = 0$

$\Rightarrow$ dif naar x_3 $\quad \frac{d\varphi_1}{dx_3}\sin\varphi_2 + \varphi_1\cos\varphi_2\frac{d\varphi_2}{dx_3} + \frac{d\varphi_2}{dx_3}\sin x_3 + \varphi_2\cos x_3 = 0$

$2\frac{d\varphi_1}{\partial x_3}\varphi_1 + \frac{d\varphi_1}{dx_3}\cdot x_3 + \varphi_1 = 0$

$\Rightarrow (1,0,0) \quad \frac{d\varphi_2}{dx_3} + 0 = 0 \quad \Rightarrow \frac{d\varphi_2}{dx_3} = 0$

$2\frac{d\varphi_1}{dx_3} + 1 = 0 \quad \frac{d\varphi_1}{dx_3} = -\frac{1}{2}$

2e orde afgeleiden:

$\frac{d^2\varphi_1}{dx_3^2}\sin\varphi_2 + 2\frac{d\varphi_1}{dx_3}\cos\varphi_2\frac{d\varphi_2}{dx_3} - \varphi_1\cdot\sin\varphi_2\left(\frac{d\varphi_2}{dx_3}\right)^2 + \varphi_1\cos\varphi_2\cdot\frac{d^2\varphi_2}{dx_3^2}$

$+ \frac{d^2\varphi_2}{dx_3^2}\sin x_3 + 2\frac{d\varphi_2}{dx_3}\cos x_3 - \varphi_2\sin x_3 = 0$

$2\frac{d^2\varphi_1}{dx_3^2}\varphi_1 + 2\left(\frac{d\varphi_1}{dx_3}\right)\left(\frac{d\varphi_1}{dx_3}\right) + \frac{d^2\varphi_1}{dx_3^2}\cdot x_3 + \frac{d\varphi_1}{dx_3} + \frac{d\varphi_1}{dx_3} = 0$

$\Rightarrow \frac{d^2\varphi_1}{dx_3^2}(2+0) + 2\cdot\left(-\frac{1}{2}\right)^2 - 1 = 0 \quad \frac{d^2\varphi_1}{dx_3^2} = \frac{1}{4}$

II

2 $f(x_1;x_2) = e^{x_1-1}\ln(x_1x_2)$

$\partial_1 f = e^{x_1-1}\ln(x_1x_2) + e^{x_1-1}\cdot\frac{1}{x_1}$ $\quad (1,1) \Rightarrow \quad 1$

$\partial_2 f = e^{x_1-1}\frac{1}{x_2}$ $\quad \Rightarrow \quad 1$

$\partial_1^2 f = e^{x_1-1}\ln(x_1x_2) + 2e^{x_1-1}\cdot\frac{1}{x_1} + e^{x_1-1}\cdot\frac{-1}{x_1^2}$ $\quad \Rightarrow \quad 1$

$\partial_1\partial_2 f = \partial_2\partial_1 f = e^{x_1-1}\cdot\frac{1}{x_2}$ $\quad \Rightarrow \quad 1$

$\partial_2^2 f = e^{x_1-1}\cdot\frac{-1}{x_2^2}$ $\quad \Rightarrow \quad -1$

$$\Rightarrow f(x_1,x_2) = 0 + 1(x_1-1) + 1(x_2-1) + \frac{1}{2}\cdot 1(x_1-1)^2 + \frac{1}{2}\cdot 2\cdot 1(x_1-1)(x_2-1)$$
$$+ \frac{1}{2}\cdot -1(x_2-1)^2 + o(\|(x_1,x_2)-(1,1)\|^2)$$

$\partial_\omega f(a) = \frac{1}{\sqrt{2}}\cdot 1 + \frac{1}{\sqrt{2}}\cdot 1$ (richting van grad(f))

3 $f: \mathbb{R}^2 \to \mathbb{R}$

$f(x_1,x_2) = x_1^6 - 3x_1^2x_2 + x_2^3$

a $\partial_1 f = 0 \qquad 6x_1^5 - 6x_1x_2 = 0 \qquad 6x_1(x_1^4 - x_2) = 0$

$\partial_2 f = 0 \qquad -3x_1^2 + 3x_2^2 = 0 \qquad -3(x_1-x_2)(x_1+x_2) = 0$

$x_2 = x_1 \Rightarrow 6x_1^2(x_1^4-1) = 0 \qquad x_1 = 0 \qquad x_1 = \pm 1$

$x_2 = -x_1 \Rightarrow 6x_1^2(x_1^4+1) = 0 \qquad x_1 = 0$

$\Rightarrow (0,0), (1,1), (-1,-1)$

b $\partial_1^2 f = 30x_1^4 - 6x_2 \qquad H_f = \begin{vmatrix} 30x_1^4 - 6x_2 & -6x_1 \\ -6x_1 & 6x_2 \end{vmatrix}$

$\partial_1\partial_2 f = -6x_1$

$\partial_2^2 f = 6x_2$

III

$(0,0)$ $H_f = 0$ geen uitspraak

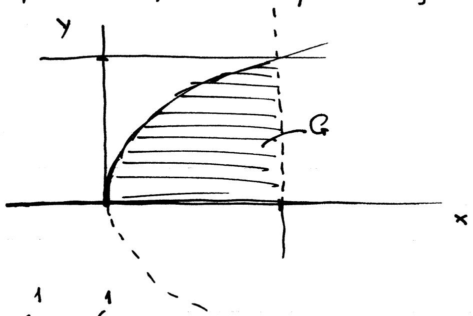
$(1,1)$ $H_f = \begin{vmatrix} 30-6 & -6 \\ -6 & 6 \end{vmatrix} = 24.6 - 6.6 = 18.6 > 0$ locaal minimum

$(-1,-1) \Rightarrow H_f = \begin{vmatrix} 30+6 & 6 \\ 6 & -6 \end{vmatrix} < 0$ zadelpunt

(c) neen t.g.v. x_2^3 is dominerend $\to \pm\infty$.

4. $G = \{(x,y) \mid 0 \le y \le 1 \quad y^2 \le x \le 1\}$

a $x = 1$
$x = y^2$
$y = 0$
$y = 1$



b $I = \int_0^1 \left[\int_{y^2}^1 y \frac{\sin x}{x} \, dx \right] dy$

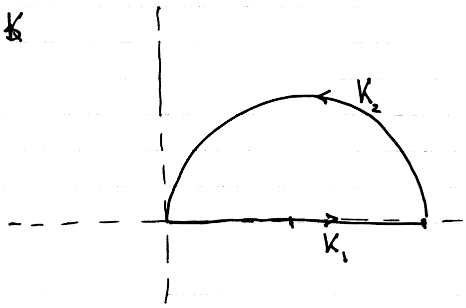
$= \int_0^1 \left[\int_0^{\sqrt{x}} y \frac{\sin x}{x} \, dy \right] dx \quad (*)$

c We berekenen $(*)$

$I = \int_0^1 \frac{1}{2} y^2 \frac{\sin x}{x} \Big|_{y=0}^{y=\sqrt{x}} dx = \int_0^1 \frac{1}{2} x \frac{\sin x}{x} \, dx = \frac{-1}{2} \cos x \Big|_0^1$

$= -\frac{1}{2} \cos 1 + \frac{1}{2} \quad (>0)$

5 K



IV

a $\int_{K_1} \underline{v}\cdot d\underline{r}$: $x_1 = t \quad (t: 0 \to 2)$
$x_2 = 0$

$$= \int_0^2 (0.1 + t.0)\,dt = 0$$

$\int_{K_2} \underline{v}\cdot d\underline{r}$: $x_1 - 1 = \cos t \quad (t: 0 \to \pi)$
$x_2 = \sin t$

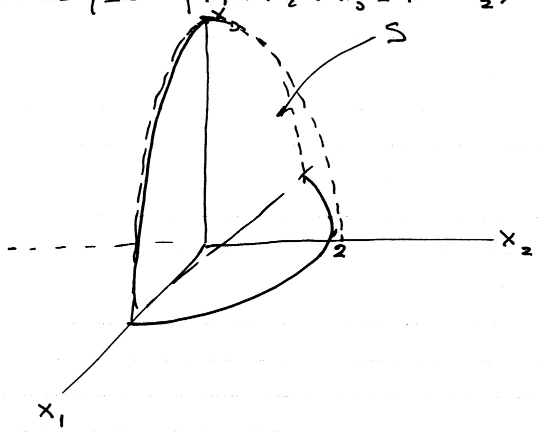
$$\int_0^\pi -\sin t\cdot(-\sin t) + (\cos t + 1)\cos t\,dt$$

$$= \int_0^\pi \underbrace{\sin^2 t + \cos^2 t}_{1} + \cos t\,dt = \pi + \sin t\Big|_0^\pi = \pi$$

b $\int_{K_1} \underline{v}\, d\underline{r} + \int_{K_2} \underline{v}\cdot d\underline{r} = \iint_V 2\, dO = 2\cdot\frac{\pi}{2} = \pi \quad \square.$

6 $S = \{\underline{x} \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3 = 4 \quad x_2 \geq 0 \quad x_3 \geq 0$

a



$x_3 = 4 - x_1^2 - x_2^2$
(paraboloïde)

b parameteriseer oppervlak

$x_1 = r\cos\varphi \qquad \varphi: 0 \to \pi$
$x_2 = r\sin\varphi \qquad r: 0 \to 2$
$x_3 = 4 - r^2$

V

$\underline{x}_r = (\cos\varphi, \sin\varphi, -2r)$

$\underline{x}_\varphi = (-r\sin\varphi, r\cos\varphi, 0)$

"n": $\underline{x}_r \times \underline{x}_\varphi = (2r^2\cos\varphi, 2r^2\sin\varphi, r)$ 3^e component positief

$\omega = (r\cos\varphi, r\sin\varphi, 4-r^2)$

$$\iint_S \omega . n\, dA = \int_0^2\int_0^{\pi} (\omega, \underline{x}_r \times \underline{x}_\varphi)\, d\varphi\, dr =$$

$$= \int_0^2 \left[\int_0^{\pi} 2r^3\cos^2\varphi + 2r^3\sin^2\varphi + 4r - r^3\, d\varphi \right] dr =$$

$$\int_0^2 \left[\int_0^{\pi} (r^3 + 4r)\, d\varphi \right] dr = \pi\left(\frac{1}{4}r^4 + 2r^2\right)\Big|_0^2 = \pi(4+8) = 12\pi$$

(c) Indien we Gauss willen toepassen moeten we S "afsluiten" met grondvlak ($x_3 = 0$) S_1 en linkerzijvlak $x_2 = 0$ S_2

S_1 : normaal $(0,0,-1)$ $\omega : (x_1, x_2, 0) \Rightarrow$ bijdrage 0

S_2 : normaal $(0,-1,0)$ $\omega : (x_1, 0, x_3) \Rightarrow$ bijdrage 0

$$\Rightarrow \iint_S \underline{\omega} . \underline{n}\, dA = \iiint_V (1+1+1)\, dx_1 dx_2 dx_3$$

oplossen met cylindercoordinaten:

$x_1 = r\cos\varphi$
$x_2 = r\sin\varphi$ $\quad |J| = r$
$x_3 = z$

$$\Rightarrow \iiint_V 3\,dx_1 dx_2 dx_3 = \int_0^2\int_0^{\pi}\int_0^{4-r^2} 3r\, dz\, d\varphi\, dr$$

$$= \pi \int_0^2 3r(4-r^2)\, dr = \pi\left(\frac{12}{2}r^2 - \frac{3}{4}r^4\right)\Big|_0^2 = \pi(24 - 12) = 12\pi \quad \square$$