Kenmerk : Vellekoop/InvTh/03
Datum : 25 maart 2005

Course : **Answers to Final Examination Introduction to Investment Theory**
Code : 151560
Date : November 26, 2003

1. a. Since

$$\begin{aligned} \frac{10000}{(1+\frac{1}{2}s_1)^2} &= 9564.74 \\ \frac{100000}{(1+\frac{1}{2}s_{0.5})^1} &= 98280.10 \end{aligned}$$

   we find that $s_{0.5} = 3.500\%$ and $s_1 = 4.500\%$

   b. We must have that

$$(1+\tfrac{1}{2}f)(1+\tfrac{1}{2}s_{0.5}) = (1+\tfrac{1}{2}s_1)^2$$

   so $f = 5.505\%$

   c. Since

$$\frac{p_f}{(1+\frac{1}{2}s_{0.5})} = \frac{10000}{(1+\frac{1}{2}s_1)^2} = 9564.74$$

   we find $p_f = 9732.12$.

   d. $D_{\text{portfolio}}$ will always be in between D_A and D_B so it can never equal $D_{\text{obligations}}$.

   e. For example

$$Q = \frac{0.5 \cdot 20000(1+\frac{1}{2}s_{0.5})^{-2} + 1.0 \cdot 200000(1+\frac{1}{2}s_1)^{-3}}{20000(1+\frac{1}{2}s_{0.5})^{-1} + 200000(1+\frac{1}{2}s_1)^{-2}}$$

   f. $\Delta P = -Q \cdot P \cdot 0.01$.

2. a. When investing percentage α in X, portfolio variance is $\alpha^2(0.05)^2+2\alpha(1-\alpha)\frac{1}{4}(0.05)(0.10)+(1-\alpha)^2(0.10)^2$ so minimizing with respect to α gives $\frac{7}{8}$ investment in X and $\frac{1}{8}$ investment in Y.

   b. $\text{cov}(X,M) = \beta_X\sigma_M^2 = 0.0032$ so $\rho_{XM} = \text{cov}(X,M)/(\sigma_X\sigma_M) = 0.8$

   c. This must equal

$$\frac{\sigma_X^2 - \beta_X^2\sigma_M^2}{\sigma_X^2}$$

   which is 36%.

3. a. Vectors are $(1.25, 1.25, 0)$, $(2.5, 0, 2.5)$ and $(1, 1, 1)$ so state prices are $\psi_1 = 0.2$, $\psi_2 = 0.6$, $\psi_3 = 0.2$.

   b. Payoff vector $(0, 0, 1)$ has price $\psi_3 = 0.2$ so if price is to be 1 we should take this vector 5 times so $z = 4$.

   c. Maximizing

$$0.6\ln(1000(1-x) + 5000x) + 0.4\ln(1000(1-x))$$

   gives optimum $x = \frac{1}{2}$.

4. Obviously, there is not just one unique good answer.