

Kenmerk : Vellekoop/InvTh/03
Datum : 18 november 2003

Course : **Final Examination Introduction to Investment Theory**
Code : 151560
Date : November 26, 2003

All answers must be motivated.
You may use an electronic calculator.
Lots of success !

1. In the US Treasury Bond market (where discounting takes place *semi-annually*) we consider two bonds:

- Bond A is a 1-year zero-coupon bond with face value of 10.000,- dollars and current price 9564,74 dollars, and
- Bond B is a 6-months zero-coupon bond with face value of 100.000,- dollars and current price 98280,10 dollars.

The contract F is a forward contract which guarantees the delivery of bond A exactly 6 months from now in return for a payment of p_F dollars at that time. Assume there are no arbitrage opportunities in this market and that the rates for borrowing and the rates for lending are equal. Short-selling of bonds is **not** allowed in this market.

- a. Find the spot rates for 6 months and 12 months. All rates should be quoted on annual basis (as usual) and with an accuracy of 1 basispoint or less.
- b. A bank offers a Forward Rate Agreement (FRA) to invest money 6 months from now for a period of 6 months. The quoted forward rate is 5.45%. Calculate the theoretical forward rate for that period, and use this to comment on the FRA rate quoted.
- c. Find the value p_F .

Somebody has to pay 2.000.000,- dollars in exactly 2 years time and he would like to create a portfolio consisting of bonds A and B as an investment for this. Since he wants the investment to be insensitive to term structure changes, he would like to immunize his portfolio against interest rate risk.

- d. Explain why it is impossible to create such a portfolio in this case.
- e. Explain how we can calculate the quasi-modified duration of a portfolio consisting of 2 bonds A and 2 bonds B (formulas are enough, you do not need to calculate an explicit value).
- f. Explain how you could use the answer in e. to estimate how the value of a portfolio consisting of 2 bonds A and 2 bonds B would react to a parallel shift for all the spot rates of +1 basispoint.

2. We consider two assets, X and Y, in a stock market which contains many other assets as well. The rates of return of X and Y have mean values $\bar{r}_X = 3\%$ and $\bar{r}_Y = 6\%$ respectively, and standard deviations $\sigma_X = 5\%$ and $\sigma_Y = 10\%$ respectively. The correlation coefficient between the rates of return is $\rho_{XY} = \frac{1}{4}$. The market portfolio asset M has a standard deviation of 8% and the beta of asset X equals $\beta_X = \frac{1}{2}$. Assume that all the assumptions of the CAPM model and Markowitz Portfolio Theory apply.

- a. Find the portfolio, consisting of assets X and Y only, which has minimal variance.
- b. Find the correlation coefficient ρ_{XM} between the rates of return of X and M.
- c. Find the percentage of variance in the rate of return of X which is firm-specific.

3. At the horse races, one can bet on three horses, *Dutch Delight (DD)*, *Dutch Courage (DC)* and *British Delight (BD)*, which have rather obvious nationalities. Only one of the horses will be the winner. For one euro you can bet (possibility A) that a Dutch horse wins (if you are wrong, you lose your euro, if you are right you get back your euro plus 0.25 euro extra) or you can bet (possibility B) that one of the 'Delight' horses wins (if you are wrong you lose your euro, if you are right you get back your euro plus 1.50 euro extra). Obviously, you can also leave some money in your pocket (possibility C).
- Find the payoff vectors for A,B and C and use these to calculate the state prices $\psi_{DD \text{ wins}}$, $\psi_{DC \text{ wins}}$ and $\psi_{BD \text{ wins}}$.
 - Suppose somebody brings a new bet on the market (possibility D), in which you bet one euro on "British Delight wins". If you are wrong you lose the euro, if you are right you get your euro back and z euro extra. Assuming that there is no arbitrage in the betting market, find¹ z .
 - An Englishman thinks that British Delight has a probability of 60% of winning, and he decides to bet part of his current wealth of 1000,- euro on that horse. He will put $1000x$ euro on British Delight and leave $1000(1-x)$ euro in his pocket, with $0 \leq x \leq 1$. Assuming that he has a logarithmic utility function and that British Delight does indeed have a probability of 60% of winning, find² his optimal value for x .
4. Suppose you are an investment banker working for the Treasury of a large corporation. You are responsible for the investment policies in bonds and other fixed income instruments which are used to pay off all sorts of liabilities in the future. Your boss, a senior manager in this corporation, is worried about the fact that interest rates may rise in the near future, since somebody has told him that rising interest rates may result in decreasing bond values. He asks you to write a short memo to him on this issue, in which you should explain what the corporation can do to protect itself against interest rate risk.

Write such a memo. Assume that your manager does not know as much as you about interest rate risk, so explain things as clearly as possible, without too much prior knowledge. The text should not be longer than three quarters of an A4 page, and it should mention the terms immunization, quasi-modified duration, and swaps.

Points:

1	a : 3	2	a : 4	3	a : 3	4	4
	b : 3		b : 2		b : 2		
	c : 3		c : 3		c : 3		
	d : 2						
	e : 2						
	f : 2						

Total: $36 + 4 = 40$ points

¹If you did not find the answer to a., use the values 0.1, 0.8 and 0.1 for the state prices.

²If you did not find the answer to b., assume that $z = 3$.