Kenmerk : Vellekoop/InvTh/TentamenJan07
Datum : 20 januari 2007

Course : **Exam Introduction to Investment Theory**
Code : 151560
Date : January 25, 2007

All answers must be motivated.
You may answer in Dutch or in English.
You may use an electronic calculator.
Lots of success !

1. Let S be an ordinary swap with 2 payment dates; one is exactly 6 months from now and one is exactly one year from now. The holder of the swap pays a fixed swap rate s and receives the short rate for the period before the payment date.

   The discount rates for 6 months, 12 months and 24 months are

   $$0.98765, \quad 0.97498, \quad 0.94731$$

   The discount rate for 18 months is unknown.

   a. Calculate the fair swap rate s (on an annual basis) with an accuracy of 1 basispoint.

   A large investment bank in the USA introduces a new financial product, the 'Swap Stepper'. For a given notional amount N this product pays a coupon every six months, for a total of 4 coupons with the first payment starting exactly 6 months from now. The first two coupons pay a fixed rate of 6% exactly and the third and fourth coupon equal the short rate for the period before those coupon dates.

   b. Show that the 'Swap Stepper' can be replicated using a floating rate bond, a swap contract[1] and a zero coupon bond by constructing a replicating portfolio for the product.[2]

   c. Find the price P of the 'Swap Stepper' as a percentage of the notional N.

2. Somebody would like to calculate the efficient frontier for portfolios consisting of a certain class of assets. He assumes all assets S_i have a normal distribution which mean μ_i and standard deviation σ_i which are known.

   a. It this enough information to calculate the efficient frontier ? Explain your answer.

   Somebody decides to invest an amount of x euros today in one particular stock. On the first of January of every year he will then look at his portfolio. If his rate of return over the past year is larger than 10%, he will stop investing. If his rate of return is smaller than 10%, he will invest an additional amount of x euros in the same stock and continue this investment for the next year. The riskfree rate of interest and the mean and variance for the normal distribution of the yearly stock returns are known, and assumed to be the same for all future years. The investor would now like to find the value of x which will optimize the expected value of the discounted profit he will make by this strategy. He intends to use the dynamic programming algorithm for this.

   b. Can the dynamic programming algorithm be used to solve this problem ? Explain your answer.

3. We use a very simple model for the unknown future of the stock market in the coming year, in which there are only two possibilities: stocks go up or stocks go down. Information about two stocks, ABC and XYZ, is given below. The probability that stocks go up equals 0.70.

[1] Which need not be the swap contract S defined above, of course.
[2] Note: you are allowed to go long or short when investing.

Product	Price Today	Price in 1 year if stocks go up	Price in 1 year if stocks go down
XYZ	99	120	90
ABC	9	11	8

a. An investor with a logarithmic utility function wants to invest exactly 1000 euros in these 2 stocks. Show how to calculate the amount of euros x he should invest in the first stock if he wants to optimize the expected utility of his payoff after 1 year (you do not need to determine the value of x).

b. Determine the riskneutral probabilities for the stock market going up and down, and the riskfree rate of interest.

c. Calculate the correct price of a European put option on XYZ with a maturity of 1 year and strike price of 110.

The Market portfolio in the stock market given above has the following characteristics.

Product	Price Today	Price in 1 year if stocks go up	Price in 1 year if stocks go down
M	17.18	20.91	15.45

If you would calculate the rates of return for the assets M and XYZ you would find that

$$r_{XYZ} = 0.9545\, r_M \; + \; 0.0455\, r_f$$

where r_f is the riskfree rate.

d. Determine the β-value of asset XYZ.

e. Determine $\bar{r}_M$ and $\bar{r}_{XYZ}$.

f. According to CAPM theory[3], does the asset XYZ perform better, worse or exactly as expected ?

Points:

1 a : 3 b : 3 c : 4	2 a : 3 b : 3	3 a : 4 b : 3 c : 3 d : 3 e : 3 f : 4	

Total: 36 + 4 = 40 points

[3] If you did not find an answer to d. use $\beta_{XYZ} = 1.0$