Kenmerk : Vellekoop/InvTh/TentamenNov07
Datum : 31 oktober 2007

Course : **Exam Introduction to Investment Theory**
Code : 151560
Date : November 7, 2007

**All answers must be motivated.**
**You may answer in Dutch or in English.**
**You may use an electronic (non-programmable) calculator.**
**Lots of success !**

1. Assume that in the current US Treasury markets the short rates for 6, 12 and 18 months (quoted on an annual basis) equal

$$r_0 = 3.00\%, \qquad r_1 = 3.50\%, \qquad r_2 = 3.75\%.$$

All discounting in this market should be done semi-annually. Let $s$ be the fair swap rate for a swap (quoted on a yearly basis), where the holder pays the floating rate and receives the fixed rate $s$ every 6 months, and which has two payments left with the the first one exactly 6 months from today.

   a. Explain carefully why we can immediately say without any calculations that

$$0.0300 < s < 0.0350.$$

   b. Calculate $s$ with an accuracy of 0.1 basispoint.

   c. Find the price of a bond which pays semi-annual coupons of 6% (quoted on a yearly basis) starting exactly 6 months from now, and which has a maturity of 18 months.

   d. Find the futures price for delivery of this bond exactly 6 months from now (immediately after it has paid its coupon for that date), under the assumption that all interest rates are deterministic.

   e. Suppose we observe a sudden parallel shift upwards in the term structure today. Will this change the fair swap rate $s$ ?

2. A new contract is announced which allows the holder to act as one of the insurers of a large ship full of oil. No premium is asked at the initial time of the contract. If the ship arrives in a certain foreign harbor safely exactly one year from now the investor receives 100 euro, but if the ship does not make it he has to pay 500 euro at that time. The probability that the ship will not make it is assumed to be 5%. The interest rate is also 5% per year.

   a. A private investor with a logarithmic utility function who currently holds 10.000 euro would like to invest part of his money in these contracts, and the rest in the bank account. Calculate how many contracts he should take if he wants to optimize the expected utility of his wealth one year from now.

   b. Find the riskneutral probability that the ship with cargo will make it safely to the foreign harbor.

   c. Somebody would like to price a new contract, where you have to pay a certain amount $p$ now and one year later you receive 1.000 if the ship arrives safely, while you get your money back if it doesn't. Find the correct price $p$.

3. Two assets $A$ and $B$ are such that the standard deviations of their rates of return per year $\sigma_A$ and $\sigma_B$ satisfy

$$\sigma_A = 2\sigma_B > 0$$

while the correlation coefficient between these rates equals $\rho_{AB} = \frac{1}{2}$. You want to create a portfolio in which you only invest in assets $A$ and $B$, and which is chosen in such a way that the variance of its yearly rate of return is minimal. You do not care about the mean rate of return.

a. How much should you invest in assets $A$ and $B$ to achieve this ?

b. Somebody with a certain given utility function asks you to find the best portfolio to maximize her expected utility one year from now if she only wants to invest in assets $A$ and $B$. Explain why it is impossible to do so based on the information given above.

4. Suppose we have $n$ different assets with yearly rates of return $r_i$ which all have a Jensen Index equal to zero: $J_i = 0$ for all $i = 1...n$.

a. Prove that any portfolio consisting of these assets also has a Jensen Index equal to zero.

The Capital Asset Pricing Model (CAPM) can be derived from Markowitz portfolio theory under a number of assumptions.

b. Discuss which assumptions need to be made for Markowitz Portfolio theory and CAPM, and indicate how realistic these assumptions are in practice.

**Points:**

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| | c | : | 3 | | c | : | 2 | | | | | | | | |
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**Total:** $36 + 4 = 40$ points