Course : **Exam Introduction to Investment Theory**
Code : 151560
Date : January 24, 2008

All answers must be motivated.
You may answer in Dutch or in English.
You may use an electronic (non-programmable) calculator.
Lots of success !

1. Consider the following zero coupon bond quotes (discount rates) with maturity 1 year, 2 years and 3 years:

$$
\begin{aligned}
D(0,1) &= 0.975610 \\
D(0,2) &= 0.954608 \\
D(0,3) &= 0.926804
\end{aligned}
$$

All discounting is done once a year i.e. on a yearly basis. Calculate

   a. The price of a bond with yearly payments of a 4% coupon, a face value of 1000, with a maturity of 2 years and its first payment exactly one year from now.

   b. The forward price for the bond mentioned in a. if the corresponding forward contract specifies delivery exactly 1 year from now, directly after the coupon payment for that date.

   c. The fair swap rate for a swap which has exactly three payments left (1 year, 2 years and three years from now). Give your answer with an accuracy of 0.1 basispoint.

   d. The duration (in years) of the bond mentioned in a.

   e. Give an equation that can be used to calculate the yield λ of the bond mentioned in a. (you do not need to solve the equation, just state it).

   We now define a forward swap which is a swap starting exactly one year from now, i.e. with payments taking place two years from now and three years from now. The fair swap rate is chosen again in such a way that today's value of the forward swap contract equals zero.

   f. Find the fair swap rate for this forward swap.

2. Consider a forward contract and a future contract on the same bond, and with the same delivery time.

   a. Explain in detail the differences between a future and a forward contract.

   b. Prove that the futures price and the forward price are the same when interest rates are deterministic.

3. A large investment bank introduces a new product, called the 'AEX Swap'. If you take one contract of this product you pay or receive nothing today but exactly one year from now you either

   • Receive 500 euro if the AEX closing price on that day is at least 10% higher than today, or

   • Pay 200 euro if the AEX closing price on that day is not at least 10% higher than today.

   The riskfree interest rate for 1 year is 5%. The market is free of arbitrage.

a. Determine the state prices and riskneutral probabilities for the event 'the AEX closing price one year from now is at least 10% higher than today' and for the event 'the AEX closing price one year from now is not at least 10% higher than today'.

Due to the huge enthusiasm for this product, the bank decides to introduce yet another product, the 'AEX Digital Call'. You pay 1000 euro for this product today. If the AEX closing price one year from now is at least 10% higher than today you receive the amount x, but if it is not, you receive nothing.

b. Find the correct value of x.

c. We have sold 100 contracts of the AEX Digital Call, and we want to replicate their payoff using AEX Swap contracts and our riskfree bank account. Find the correct number of AEX Swap Contracts and the correct amount of money to put on the bank account to make sure that we have a riskless position.

4. Suppose you have a logarithmic utility function and your current wealth is 5000 euro. You are given two betting possibilities: the first bet gives you a 5% chance of winning 200 euro (and you receive nothing otherwise) and the second one gives you a 2% chance of winning 500 euro (and you receive nothing otherwise). Which bet should you take ?

Points:

1			2			3			4		
	a	: 2		a	: 4		a	: 3		:	4
	b	: 3		b	: 4		b	: 3			
	c	: 2					c	: 3			
	d	: 2									
	e	: 2									
	f	: 4									

Total: $36 + 4 = 40$ points