Kenmerk : Vellekoop/InvTh/TentamenNov08
Datum : 30 oktober 2008

Course : **Exam Introduction to Investment Theory**
Code : 151560
Date : November 5, 2008

All answers must be motivated.
You may answer in Dutch or in English.
You may use an electronic (non-programmable) calculator.

Short-selling is always allowed.

Lots of success !

1. Consider a market with many assets, among which there is a riskfree asset with yearly rate of return $r_f = 3\%$, a market asset M with stochastic yearly rate of return r_M and an asset A with stochastic yearly rate of return r_A. The standard deviations of r_M and r_A per year are

$$\sigma_M = 3\%, \qquad \sigma_A = 5\%,$$

while their means are

$$\bar{r}_M = 12\%, \qquad \bar{r}_A = 15\%.$$

We assume that this market follows the CAPM model perfectly.

   a. Determine the β value of asset A.
   b. Calculate the correlation coefficient between r_A and r_M.
   c. Determine what percentage of the variance of r_A is company-specific.

An investor wants to invest all his money in assets A and assets M, but in such a way that his portfolio rate of return has minimal variance.

   d. Determine how his money should be invested[1].

2. We model the price of a stock S next year using three possible states, in which the stock will have the value 16, 12 or 8 euro. Today's stock value equals 10.40 euro. We also consider a contract D which is a **digital call** on the stock; it costs 5 euro today and will pay 20 euro if the stock is worth 16 next year, and it will pay zero otherwise. A third contract C is a call on the stock S with a maturity of 1 year and a strike of 11 euro, which costs 1.45 euro today.

   a. Determine the state prices and riskneutral possibilities of the three possible states.
   b. Determine the riskfree rate r_f in this model.
   c. Show how 1000 contracts C can be hedged using contracts S, contracts D and a bankaccount which earns the riskfree rate r_f.

Somebody has a current wealth of 500 euro and he wants to optimize his expected utility of wealth one year from now by investing an amount $x \in [0, 500]$ euro in contract D. This person has the utility function $U(w) = \sqrt{w}$ and he believes that the probability that the stock price of S will go up to 16 euro next year equals 80%.

   d. Determine the optimal value of x for this investor.

[1] If you did not find an answer to b. then use $\rho = \frac{1}{3}$.

3. Consider a term structure of interest where discounting happens once a year, and where the one-year short rates for the coming six years are (a, a, a, b, b, b) i.e. the short rates for the first three years are the same and the short rates for years four to six are the same too. Obviously, a and b need not be the same.

   a. Give a formula for today's price P of an annuity which pays a yearly fixed amount A for the coming six years, i.e. the first payment is exactly one year from now and there are six payments in total.

   b. Find a formula for the swap rate s for a three-year swap where we receive floating and pay fixed every year. The first payment day is exactly one year from now and we make three payments in total.

   c. Somebody would like to arrange a forward rate agreement with us for money which is invested exactly two years from now, and paid back exactly four years from now. Find the appropriate deal rate for such an agreement.

   We want to construct a portfolio of bonds which allows us to pay for a liability of 1 million euro exactly four years from now. All bonds that we can use for our portfolio have maturities which are equal to or less than six years. We are very worried about a parallel shift in the rates for the first three years (that is, a change in a) but also about a parallel shift in the rates for the last three years (a change in b). Assume that these are the only interest rate changes that can occur.

   d. Describe **in detail** how we can construct a bond portfolio to pay for our liability which is protected against *both* forms of parallel shifts.

Points:

1			2			3			
	a	2		a	3		a	3	
	b	3		b	2		b	2	
	c	3		c	4		c	2	
	d	4		d	4		d	4	

Total: $36 + 4 = 40$ points