

Kenmerk : Vellekoop/InvTh/TentamenJan09
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Course : Final Exam Introduction to Investment Theory
Code : 151560
Date : January 22, 2009

All answers must be motivated.
You may use an electronic calculator.
Markets are free of arbitrage unless stated otherwise.

Lots of success !

1. At the end of 2008 it was revealed, in the wake of the credit crisis, that former NASDAQ topman Bernard Madoff had run a fraudulent investment fund which was believed to have destroyed 50.000.000.000 US dollars. Investigations into the scandal brought to light that the returns which the fund had reported had been very unrealistic and that anonymous warnings had therefore been sent to US regulators by certain investment experts.

The data of the rates of return of Madoff's fund r^{fund} and the market rates of return r^M are known now. Assume that we have yearly rate of return data $(r_t^{\text{fund}}, r_t^M)$ for the years $t = N, N + 1, \dots, M$ for some $N < M$.

- a. Explain *in detail* how the β -value of the fund can be estimated from these data.

Estimates of the yearly mean rates of return of the fund and of the market turn out to equal $\bar{r}^{\text{fund}} = 12.00\%$ and $\bar{r}^M = 8.64\%$ respectively. The β of the fund was estimated to be $\beta = 0.06$ and the average yearly riskfree rate of return during the investigated period can be estimated to be $r_f = 4.96\%$.

- b. Calculate the Jensen Index for the fund.
- c. Discuss the fund's performance using the Capital Asset Pricing Model. Motivate your answer by discussing the CAPM assumptions in detail.

Madoff claimed that the risk in his fund was decreased by his trading in certain options. If he had a certain stock in his fund with a current price S , he would regularly shortsell call options with strike $K_c > S$ and use the money to buy put options with strike $K_p < S$.

- d. Explain why this reduces the risk in the portfolio.
 - e. Explain whether Capital Asset Pricing is still the correct model to use if apart from stocks we also add options to our portfolio. If it is, explain how options can be incorporated into the CAPM model. If not, explain why not.
2. There is a growing market now for so called *weather derivatives*. Consider the following three possible events:
 - A On January 22, 2010 the official average temperature in Holland is strictly larger than 20 degrees celsius
 - B On January 22, 2010 the official average temperature in Holland is strictly less than 10 degrees celsius
 - C On January 22, 2010 the official average temperature in Holland is in between 10 and 20 degrees celsius (including 10 and 20 degrees celsius)

Suppose that someone offers three weather contracts: contract A pays 900,- euro in case event A happens (and nothing otherwise), contract B pays 600,- euro in case event B happens (and nothing otherwise) while contract C pays 300,- euro in case event C happens (and nothing otherwise). These three contracts all have the same price today (January 22, 2009) of p euro. Assume that the riskfree rate in Holland is constant at 10% yearly.

- a. Determine p .
- b. Find the *riskneutral* probability of event B .
- c. Explain carefully why you do not need the *real* probabilities of the three events happening to find the correct price p .

Assume that a certain investor with a logarithmic utility function wants to invest all his money in these weather contracts A , B and C (he does *not* want to invest in riskfree assets!). He has 18.000,- euro to invest and he wants to do it in such a way that his expected utility after one year is maximized. Assume that the *real* probabilities of the events A , B and C are $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{2}$ respectively.

- d. Calculate the optimal investment portfolio for this investor.¹
- e. Determine the certainty equivalent value of this investment.²

3. Consider an annuity product which pays a fixed coupon of A dollars on January 22 of every year in the future, with the first payment exactly one year from now (on January 22, 2010). Assume that the price of this 'infinite-life' product is P dollars today and that the term structure is flat, so all yields λ are the same for all future maturities.

- a. Determine the yield λ of the product in terms of A and P .
- b. Determine the modified duration of this product in terms of A and P .
(Hint: calculate $\frac{dP}{d\lambda}$).

Today a new forward contract is created in this market which specifies that one annuity product as described above must be bought for a price of F on January 23, 2012 (i.e. 3 years from now but directly after the payment for that year). As usual, the price of this new forward contract is zero today.

- c. Determine the correct forward price F in terms of A and P .

Points:

1	a : 4	2	a : 4	3	a : 3
	b : 1		b : 2		b : 3
	c : 3		c : 2		c : 4
	d : 2		d : 4		
	e : 2		e : 2		

Total: $36 + 4 = 40$ points

¹If you did not find an answer for question a. you may assume that $p = 100$.

²If you did not find an answer for question d. you may assume that he invests equal amounts in all three assets.