Course : **Final Examination Introduction to Investment Theory**

Code : 191515603

Date : November 3, 2010

All answers must be motivated
You may use an electronic calculator
Answers may be written in English or Dutch
Lots of success !

1. Two bonds X and Y, both of which mature in exactly two years from now, have face values of 100. Coupons are paid annually. A coupon payment has just been made in both cases. Bond X has a coupon rate of 13% and stands at a price of 105. Bond Y has a coupon rate of 5% and stands at a price of 91.25.

   (a) Calculate the 1-year and 2-year annual spot rates.

   (b) If I invest € 1000 one year from now, how much return I expect one year thereafter?

   (c) Let λ_X and λ_Y be the yields of the two bonds. Calculate their durations.

2. An investor intends to construct a portfolio P consisting entirely of a mixture of assets A and B. Short-selling is not allowed. Let r_A, r_B and r_P denote the rates of return of A, B and P respectively. We are given the following estimates: $\bar{r}_A = 0.10$, $\bar{r}_B = 0.20$, $\sigma_A = 0.20$ and $\sigma_B = 0.30$. Assuming that our objective is the minimization of the standard deviation σ_P of our portfolio, what proportion of available funds should be invested in A and what proportion in B if the rates of return of A and B are:

   (a) perfectly positively correlated ($\rho_{AB} = +1$);

   (b) uncorrelated ($\rho_{AB} = 0$);

   (c) perfectly negatively correlated ($\rho_{AB} = -1$).

3. Suppose that the spot price of gold today is € 300 per ounce and the continuously compounded risk-free rate is 5%. There are no carrying costs with holding gold but gold can be leased instead at a rate of 2% per year, assumed to be paid continuously.

   (a) What is the price of a gold futures with delivery in 6 months?
   (<u>Hint:</u> Carrying gold is actually yielding a return).

   (b) The owner of some gold would like to borrow € 30,000 today and in six months repay the loan in gold instead of euros (This is called a gold-linked note). Suppose the contract calls for the owner of the gold to deliver 100 ounce of gold in six months. What additional cash payment P needs to be made at delivery in order

for both the owner of gold and the lender to be willing to enter into the contract? (<u>Hint</u>: Set up a transaction table).

4. Consider a single-period binomial model where the "stock" is the market portfolio and $\Delta t = 1/52$ (that is, one week). Assume the volatility of the market return to be $\sigma = 20\%$ per year. A consistent way to define the binomial parameters is to set

$$u = e^{\sigma\sqrt{\Delta t}} \quad \text{and} \quad d = e^{-\sigma\sqrt{\Delta t}}.$$

The current risk-free interest rate is 5%, compounded weekly. The present price of the market portfolio is $S = €\,100$.

(a) Compute the risk-neutral probability q of the market going up.

(<u>Hint</u>: q can be calculated by making sure that the risk-neutral formula holds for the underlying stock; that is, $S = \frac{1}{R}[quS + (1-q)dS])$.

(b) Use risk-neutral valuation to compute the price V of a European option with payoff $V_u = €\,150$ and $V_d = €\,50$.

(c) Replicate the option of part (b) by forming a portfolio in the market and the risk-free bond. What is the option's beta?

(<u>Hint</u>: Betas are additive. Use the market's beta and the bond's beta to solve the problem).

Points:

1	a	:	5	2	a	:	5	3	a	:	5	4	a	:	5
	b	:	5		b	:	5		b	:	5		b	:	5
	c	:	5		c	:	5						c	:	5

Total: $55 + 5$ bonus points $= 60$ points