

Final Examination

Introduction to Investment Theory (191515603), 2012-2013

Date: 7-November-2012, 13:45 – 16:45

Full Marks : 45

All answers must be motivated

You may use an electronic calculator

Answers may be written in English or Dutch

Lots of success !

1. In the US Treasury Bond market (where discounting takes place semi-annually) we consider two bonds:

- Bond A is a 1-year zero-coupon bond with face value of 10 000 dollars and current price 9564.74 dollars, and
- Bond B is a 6-months zero-coupon bond with face value of 100 000 dollars and current price 98280.10 dollars.

The contract F is a forward contract which guarantees the delivery of bond A exactly 6 months from now in return for a payment of p_F dollars at that time. Assume there are no arbitrage opportunities in this market and that the rates for borrowing and the rates for lending are equal. Short-selling of bonds is not allowed in this market.

- (a) Find the spot rates for 6 months and 12 months. All rates should be quoted on annual basis (as usual) and with an accuracy of 1 basis-point (bp) or less, i.e., correct up to four decimal places or more. (100 bp = 1%) [4]

[If you do not find the answers in (a), assume $s_{0.5} = 4.000\%$ and $s_1 = 4.300\%$.]

- (b) A bank offers a Forward Rate Agreement (FRA) to invest money 6 months from now for a period of 6 months. The quoted forward rate is 5.45%. Calculate the theoretical forward rate for that period, and use this to comment on the quoted FRA rate – is this a bargain or you rather stay away from this FRA? [3]
- (c) Find the value p_F of the forward contract F. [3]

Somebody has to pay 2 000 000 dollars in exactly 2 years time and he would like to create a portfolio consisting of bonds A and B as an investment for this. Since he wants the investment to be insensitive to term structure changes, he would like to immunize his portfolio against interest rate risk.

- (d) Explain why it is impossible to create such a portfolio in this case. [3]
- (e) Explain how we can calculate the quasi-modified duration of a portfolio consisting of 2 bonds A and 2 bonds B (formulas are enough, you do not need to calculate an explicit value). [2]

2. We consider two assets, X and Y, in a stock market which contains many other assets as well. The rates of return of X and Y have mean values $\bar{r}_X = 3\%$ and $\bar{r}_Y = 6\%$, respectively, and standard deviations $\sigma_X = 5\%$ and $\sigma_Y = 10\%$, respectively. The correlation coefficient between the rates of return is $\rho_{XY} = \frac{1}{4}$. The market portfolio asset M has a standard deviation of 8% and the beta of asset X equals $\beta_X = \frac{1}{2}$. Assume that all the assumptions of the CAPM model and Markowitz Portfolio Theory apply.

- (a) Find the minimum variance portfolio, consisting of assets X and Y only. [4]
- (b) Find the correlation coefficient ρ_{XM} between the rates of return of X and M. [2]
- (c) What percentage of the variance in the rate of return of X is firm-specific? [3]

3. For the Eredivisie (Football) Game between FC Twente and Feyenoord on Sunday, 4 November, 2012, the official “bwin.com” betting odds are

FC Twente wins	payoff 1.67 euro per euro bet
Feyenoord wins	payoff 4.50 euro per euro bet
Draw (nobody wins)	payoff 3.60 euro per euro bet

This means that if you bet 1 euro on ‘FC Twente wins’ then you will receive nothing if Feyenoord wins or if no team wins, but you will receive your euro back with 67 euro-cents extra if FC Twente wins.

- (a) Define a three-dimensional state price model for the bets and calculate the corresponding state prices. [3]

[If you do not find the answers in (a), assume the state prices to be 0.70, 0.15, and 0.25.]

- (b) Calculate in this model the risk-free rate of return r_f . [2]

[This rate is for the single-period between you placing the bet and you receiving the outcome. No need to convert it to yearly rate.]

- (c) Can you explain (intuitively) why the risk-free rate is negative? [2]

- (d) Calculate the risk-neutral probability that FC Twente will win the game. [2]

- (e) Somebody defines a new contract which is called ‘*FC Twente Light*’. You receive 5 euro if FC Twente wins, you receive nothing if Willem II wins, but you get your money (i.e., the price of the contract) back in case of a draw. Find the correct price for such a contract. [3]

- (f) Suppose you are a huge FC Twente fan and you believe that the probability that FC Twente wins is 70%. You have 100 euro in your pocket and decide to bet (via bwin.com) a percentage on FC Twente (the original one in the table, not *FC Twente Light*). Assume you have a logarithmic utility function. Find out how much money you should bet on FC Twente to optimize the expected utility of the amount of money you own after the game. [4]

4. You are a fund manager in a large investment bank and your boss is worried about a certain large portfolio containing several different Dutch stocks. He has heard that there are different ways to analyze portfolios: mean-variance analysis, Markowitz Models and CAPM, but he does not know the details about any of these. He therefore asks you to write a memo (maximal length: 3 quarters of a page) on this issue, in which you explain how the performance of a portfolio can be measured in an objective way.

Write this memo.

[5]

Final Grade: $\frac{\text{score on exam} + 5}{5}$ (rounded off to an integer)
