

**Make-up Examination**  
**Introduction to Investment Theory (191515603), 2012-2013**

Date: 23-January-2013, 13:45 – 16:45

Full Marks : 36

**All answers must be motivated**

**You may use an electronic calculator**

**Answers may be written in English or Dutch**

**Lots of success !**

1. A bank offers the following rates (expressed, as usual, on annual basis) to its customers. The bank makes no distinction in rates between borrowing and lending.
  - A short rate of 2.5% for 6 months.
  - A forward rate of 2.7% for investing money 6 months from now for a period of 6 months.
  - A forward rate of 2.8% for investing money one year from now for a period of 6 months.
  - (a) Find the discount rates  $d_{0,1}$ ,  $d_{0,2}$  and  $d_{0,3}$  for the coming 6, 12 and 18 month periods. All rates should be quoted correctly up to three decimal places or more. [3]  
*[If you do not find the answers in (a), assume  $d_{0,1} = 0.9896$ ,  $d_{0,2} = 0.9697$  and  $d_{0,3} = 0.96$  for the following questions.]*
  - (b) Suppose, you are interested in a swap deal with 3 payment dates, namely, 6, 12 and 18 months from now, where you pay the floating short rates of the previous period and you receive the fixed swap rate. What would be a fair swap-rate based on the information given above? [4]
  - (c) Let  $A$  be a bond with maturity 12 months from now and a face value of 100 000 euro. The annual coupon payment of 4% is paid out semiannually. Calculate the price and quasi-modified duration (expressed in years) of this bond. [2+2]  
*[If you do not find the answers in (c), assume the price 101 800 and quasi-modified duration of 0.98 years for the following questions.]*
  - (d) What will the new price of this bond be if today a parallel shift in the spot rate curve occurs which lowers all spot rates by 0.6%? [2]
2. We consider two assets, X and Y, in a stock market which contains many other assets as well. The rates of return of X and Y have mean values  $\bar{r}_X = 5\%$  and  $\bar{r}_Y = 6.8\%$ , respectively, and variances  $\sigma_X^2 = 12\%$  and  $\sigma_Y^2 = 20\%$ , respectively. The correlation coefficient between the rates of return is  $\rho_{XY} = \frac{1}{4}$ . We are interested in a portfolio of the assets X and Y where no short selling is allowed.
  - (a) Find such a portfolio which has minimum variance. Calculate the corresponding expected return and the (minimum) variance. [3+1+1]
  - (b) Present in a rough plot the efficient frontier for a portfolio made up of a risk free asset with interest rate  $r_f = 3.5\%$  and the risky assets X and Y. Once again assume that short selling of risky assets are not allowed. [2]
3. The current gold price is 500 euro per ounce and gold has an annual storage cost of 2.5 euro per ounce, payable semi-annually in advance. Suppose the (constant) risk free rate is 4% per annum compounded semi-annually, find the futures price of gold with maturity 1.5 year. [4]

4. We want to find the value of a European put option with strike price  $K = 55$  and maturity two months from now, for a stock with current price  $S(0) = 54$  and (annual) volatility  $\sigma = 0.33$ . The current (annual) interest rate is  $r = 3.5\%$ , compounded monthly.

We want to use a two-period binomial tree, with a period being a month to find the required put option value. Recall that a consistent way to define the binomial parameters is to set  $u = e^{\sigma\sqrt{\Delta t}}$  and  $d = e^{-\sigma\sqrt{\Delta t}}$ .

- (a) Compute the risk-neutral probability  $q$  of the underlying asset going up, by using the fact that risk-neutral valuation formula also holds for the asset, i.e., the current price is the discounted expected value of the price a period later, where expectation is w.r.t. the risk-neutral probabilities. [2]

*[If you do not find the answer in (a), assume  $q = 0.45$  for the following questions.]*

- (b) Construct a relevant binomial tree and based on the tree calculate the price of the European put option described above. [5]

5. Write a short memo about the possible interest rate risks to fixed income securities and how one can possibly protect oneself against them. The memo should not be longer than three quarter of an A4 page and explain things as clearly as possible. [5]

<b>Final Grade:</b> $\frac{\text{score on exam} + 4}{4}$ (rounded off to an integer)
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