SMOM uitw. 11-08-2004

(1) (a) fasen = rangnummers beurt = {0, 1, 2, 3} (1)

toestand = hoeveelheid in kas (1)

beslissingen = {stoppen of doorgaan} (1)

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fase	toestandsruimte
0	3750
1	0, 4000, 4250, 4750
2	0, 250, 500, 1000, 4250, 4500, 4750, 5000, 5250, 5750
3	0, 250, 500, 750, 1000, 1250, 1500, 2000, 4500, 4750, 5000, 5250, 5500, 5750, 6000, 6250, 6750.

(2)

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(b) Na ~~het~~ de 3e ~~spel~~beurt is het spel afgelopen en is dus x in $f_3(x)$ het eindbedrag. (2)

(c) Per definitie geldt dat

$$f_2(x) = \max \begin{cases} x & [\text{stop}] \\ \frac{4}{10} f_3(x+250) + \frac{3}{10} f_3(x+500) + \frac{2}{10} f_3(x+1000) + \frac{1}{10} f_3(0) & [\text{doorgaan}] \end{cases}$$

$$= \max \begin{cases} x \\ \frac{4}{10}(x+250) + \frac{3}{10}(x+500) + \frac{2}{10}(x+1000) + \frac{1}{10}\cdot 0 \end{cases}$$

$$= \max \begin{cases} x & [\text{stop}] \\ 0.9x + 450 & [\text{doorgaan}] \end{cases}$$

Dus

$$f_2(x) = x \quad \text{voor} \quad x \geq 0.9x + 450 \iff x \geq 4500$$

$$= 0.9x + 450 \quad \text{voor} \quad x \leq 4500$$

Ledere redenati: 3

Een "eenvoudiger" redenering:

- als je stopt is het eindbedrag x
- ga je door, dan is het gemiddelde eindbedrag

$$\frac{4}{10}(x+250)+\frac{3}{10}(x+500)+\frac{2}{10}(x+1000)+\frac{1}{10}\cdot 0 = 0.9x+450$$

Dit leidt tot dezelfde conclusie.

Nog eenvoudiger:

- de gemiddelde opbrengst van doorgaan bedraagt:

$$\frac{4}{10}\cdot 250+\frac{3}{10}\cdot 500+\frac{2}{10}\cdot 1000-\frac{1}{10}\cdot x = 450-\frac{1}{10}\cdot x$$

Dit is alleen voordelig als dit niet-negatief is, dus $x \geq 4500$.

(c)

$$f_k(x) = \max \begin{cases} x & [\text{stop}] \\ \frac{4}{10}f_{k+1}(x+250)+\frac{3}{10}f_{k+1}(x+500)+\frac{2}{10}f_{k+1}(x+1000)+\frac{1}{10}f_{k+1}(0) & [\text{doorgaan}] \end{cases}$$

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voor $k=0,1$.

~~$f_0(x) = m$~~

(d)

$$f_1(0) = \max \begin{cases} 0 \\ \frac{4}{10}f_2(250)+\frac{3}{10}f_2(500)+\frac{2}{10}f_2(1000)+\frac{1}{10}f_2(0) \end{cases}$$

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$$= \max \begin{cases} 0 \\ \frac{4}{10}\cdot\left(\frac{9}{10}\cdot 250+450\right)+\frac{3}{10}\left(\frac{9}{10}\cdot 500+450\right)+\frac{2}{10}\left(\frac{9}{10}\cdot 1000+450\right)+\frac{1}{10}\cdot 450 \end{cases}$$

$$= \max \begin{cases} 0 & [\text{stop}] \\ 855 & [\text{doorgaan}] \end{cases}$$

3

$$= 855 \ [\text{doorgaan}]$$

$$f_1(4000) = \max \begin{cases} 4000 \\ \frac{4}{10}\left[\frac{9}{10}\cdot 4250+450\right]+\frac{3}{10}\cdot 4500+\frac{2}{10}\cdot 5000+\frac{1}{10}\cdot 450 = 4105 \end{cases}$$

$$= 4105 \ [\text{doorgaan}]$$

$$f_1(4250) = \max \begin{cases} 4250 & [\text{stoppen}] \\ \frac{4}{10}\left[\cancel{\tfrac{4}{10}}4500\right] + \frac{3}{10}\cdot 4750 + \frac{2}{10}\cdot 5250 + \frac{1}{10}\cdot 450 = 4320 & [\text{doorgaan}] \end{cases}$$

$$f_1(4250) = 4320 \ [\text{doorgaan}]$$

$$f_1(4750) = \max \begin{cases} 4750 & [\text{stoppen}] \\ \frac{4}{10}\cdot 5000 + \frac{3}{10}\cdot 5250 + \frac{2}{10}\cdot 5750 + \frac{1}{10}\cdot 450 = 4770 & [\text{doorgaan}] \end{cases}$$

$$= 4770 \ [\text{doorgaan}]$$

$$f_0(3750) = \max \begin{cases} 3750 & [\text{stoppen}] \\ \frac{4}{10} f_1(4000) + \frac{3}{10} f_1(4250) + \frac{2}{10} f_1(4750) + \frac{1}{10} f_1(0) & [\text{doorgaan}] \end{cases}$$

1

$$= \max \begin{cases} 3750 \\ \frac{4}{10}\cdot 4105 + \frac{3}{10}\cdot 4320 + \frac{2}{10}\cdot 4770 + \frac{1}{10}\cdot 855 = 3977.5 \end{cases}$$

$$= 3977.5$$

Het verwachte eindbedrag bedraagt 3977.5.
De optimale politiek is om door te spelen t/m de 2^e beurt

1

en vervolgens te stoppen indien het verdiende bedrag tenminste 4500 bedraagt en anders door te gaan met de 3^e beurt.

Opgave 2.

(a)
$$f(\text{goed}) = \max \begin{cases} \overbrace{\tfrac{1}{2}\times 9 + \tfrac{1}{2}\times 3}^{6} + \tfrac{1}{2}\left[\tfrac{1}{2} f(\text{goed}) + \tfrac{1}{2} f(\text{slecht})\right] & [\text{niet adv.}] \\ \underbrace{\tfrac{8}{10}\times 4 + \tfrac{2}{10}\times 4}_{4} + \tfrac{1}{2}\left[\tfrac{8}{10} f(\text{goed}) + \tfrac{2}{10} f(\text{slecht})\right] & [\text{adv}] \end{cases}$$

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$$f(\text{slecht}) = \max \begin{cases} \underbrace{\tfrac{2}{5}\times 3 + \tfrac{3}{5}\times(-7)}_{-3} + \tfrac{1}{2}\left[\tfrac{2}{5} f(\text{goed}) + \tfrac{3}{5} f(\text{slecht})\right] & [\text{geen res.}] \\ \underbrace{\tfrac{7}{10}\times 1 + \tfrac{3}{10}\times(-19)}_{-5} + \tfrac{1}{2}\left[\tfrac{7}{10} f(\text{goed}) + \tfrac{3}{10} f(\text{slecht})\right] & [\text{res}] \end{cases}$$

(b) Er zijn vier stationaire politieken: met beslissingsregels

5 [niet adv., geen res.], [niet adv., res.],

[adv., geen res.], [adv., res.].

(c) Stel $f_1(\text{goed}) = f_1(\text{slecht}) = 0$.

$$f_2(\text{goed}) = \max \begin{cases} 6 & [\text{niet adv}] \\ 4 & [\text{adv}] \end{cases} = 6$$

3
$$f_2(\text{slecht}) = \max \begin{cases} -3 & [\text{geen res.}] \\ -5 & [\text{res.}] \end{cases} = -3$$

$$f_3(\text{goed}) = \max \begin{cases} 6 + \tfrac{1}{2}\left[\tfrac{1}{2}\times 6 + \tfrac{1}{2}\times(-3)\right] \\ 4 + \tfrac{1}{2}\left[\tfrac{8}{10}\times 6 + \tfrac{2}{10}\times(-3)\right] \end{cases} = 6\tfrac{3}{4}$$

3
$$f_3(\text{slecht}) = \max \begin{cases} \cancel{6} -3 + \tfrac{1}{2}\left[\tfrac{2}{5}\times 6 + \tfrac{3}{5}\times(-3)\right] \\ -5 + \tfrac{1}{2}\left[\tfrac{7}{10}\times 6 + \tfrac{3}{10}\times(-3)\right] \end{cases} = -2\tfrac{7}{10}$$

(d) Beschouw de stationaire politiek met als beslisregel

[adv. , geen res.] := π

3

$$\left.\begin{aligned} V_\pi(\text{goed}) &= 4 + \tfrac{1}{2}\left[\tfrac{8}{10}V_\pi(\text{goed}) + \tfrac{2}{10}V_\pi(\text{slecht})\right] \\ V_\pi(\text{slecht}) &= -3 + \tfrac{1}{2}\left[\tfrac{2}{5}V_\pi(\text{goed}) + \tfrac{3}{5}V_\pi(\text{slecht})\right] \end{aligned}\right\} \text{'waardebepaling'}$$

Oplossen van deze vergelijkingen geeft:

$$V_\pi(\text{goed}) = 6\tfrac{1}{4} \quad \text{en} \quad V_\pi(\text{slecht}) = -2\tfrac{1}{2}$$

De 'verbeteringsstap' geeft:

toestand '<u>goed</u>':

3

$$\max\begin{cases} 6 + \tfrac{1}{2}\left[\tfrac{1}{2}\times\tfrac{25}{4} + \tfrac{1}{2}\times\left(-\tfrac{5}{2}\right)\right] & [\text{niet adv.}] \\ 4 + \tfrac{1}{2}\left[\tfrac{8}{10}\times\tfrac{25}{4} + \tfrac{2}{10}\times\left(-\tfrac{5}{2}\right)\right] & [\text{adv.}] \end{cases}$$

$$= \max\begin{cases} 6\tfrac{15}{16} & [\text{niet adv.}] \longleftarrow ! \\ 6\tfrac{1}{4} & [\text{adv.}] \end{cases}$$

In toestand 'goed' moet de beslissing <u>gewijzigd</u> worden, dus is de startpolitiek π <u>niet optimaal.</u>

~~Het is niet meer nodig de toestand 'slecht' te onderzoeken!~~

toestand '<u>slecht</u>'

$$\max\begin{cases} -3 + \tfrac{1}{2}\left[\tfrac{2}{5}\times\tfrac{25}{4} + \tfrac{3}{5}\times\left(-\tfrac{5}{2}\right)\right] & [\text{geen res.}] \\ -5 + \tfrac{1}{2}\left[\tfrac{7}{10}\times\tfrac{25}{4} + \tfrac{3}{10}\times\left(-\tfrac{5}{2}\right)\right] & [\text{res.}] \end{cases}$$

$$= \max\begin{cases} -2\tfrac{1}{2} & [\text{geen res.}] \longleftarrow ! \\ -3\tfrac{13}{16} & [\text{res.}] \end{cases}$$

1 slordig

In toestand 'slecht' wordt de beslissing <u>niet</u> gewijzigd.

De stationaire verbetering die vervolgens wordt onderzocht is

[~~geen~~ niet adv. , geen res.].

(e) min $x_1 + x_2$

z.d.d.

$$x_1 \geq 6 + \frac{1}{4} x_1 + \frac{1}{4} x_2 \qquad 3x_1 \geq 24 + x_2 \quad *$$

$$x_1 \geq 4 + \frac{4}{10} x_1 + \frac{1}{10} x_2 \qquad 6x_1 \geq 40 + x_2$$

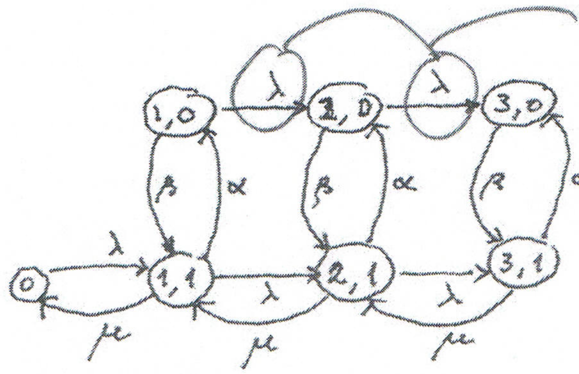
$$x_2 \geq -3 + \frac{1}{5} x_1 + \frac{3}{10} x_2 \qquad 7x_2 \geq -30 + 2x_1 \quad *$$

$$x_2 \geq -5 + \frac{7}{20} x_1 + \frac{3}{20} x_2 \qquad 17 x_2 \geq -100 + 7x_1$$

−2

Opg. 3

(a)



7

$(i,0)$ = i # in systeem, server in reparatie

$(i,1)$ = i # " " , " werkt

(b)

4

$$\lambda P_0 = \mu P_{11} \qquad (1)$$

$$(\lambda+\mu+\alpha) P_{11} = \lambda P_0 + \beta P_{10} + \mu P_{21} \qquad (2)$$

$$(\lambda+\beta) P_{10} = \alpha P_{11} \qquad (3)$$

$$(\lambda+\mu+\alpha) P_{21} = \lambda P_{11} + \beta P_{20} + \mu P_{31} \qquad (4)$$

$$(\lambda+\beta) P_{20} = \alpha P_{21} + \lambda P_{10} \qquad (5)$$

$$(\mu+\alpha) P_{31} = \lambda P_{21} + \beta P_{30} \qquad (6)$$

$$\beta P_{30} = \lambda P_{20} + \alpha P_{31} \qquad (7)$$

$$(1) \quad P_{11} = \frac{\lambda}{\mu} P_0 \xrightarrow{(3)} P_{10} = \frac{\alpha}{\lambda+\beta} \cdot \frac{\lambda}{\mu} P_0$$

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$$(1)+(2)+(3) \implies \lambda P_{11} + \lambda P_{10} = \mu P_{21}$$

$$\rightarrow P_{21} = \left(\frac{\lambda}{\mu}\right)^2 P_0 + \left(\frac{\lambda}{\mu}\right)^2 \frac{\alpha}{\lambda+\beta} P_0 = \left(1+\frac{\alpha}{\lambda+\beta}\right)\left(\frac{\lambda}{\mu}\right)^2 P_0$$

$$(5): \quad P_{20} = \frac{\alpha}{\lambda+\beta} \cdot \left(1+\frac{\alpha}{\lambda+\beta}\right)\left(\frac{\lambda}{\mu}\right)^2 P_0 + \frac{\lambda}{\lambda+\beta} \cdot \frac{\alpha}{\lambda+\beta} \cdot \frac{\lambda}{\mu} P_0$$

$$= \frac{\alpha}{\lambda+\beta} \cdot \frac{\lambda}{\mu} \cdot \left\{ \frac{\lambda}{\lambda+\beta} + \frac{\lambda}{\mu}\left(1+\frac{\alpha}{\lambda+\beta}\right) \right\} P_0$$

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$$(6)+(7): \quad \mu P_{31} = \lambda P_{20} + \lambda P_{21}$$

7/1

$$P_{31} = \left(1+\frac{\alpha}{\lambda+\beta}\right)\left(\frac{\lambda}{\mu}\right)^3 P_0 + \frac{\alpha}{\lambda+\beta}\left(1+\frac{\alpha}{\lambda+\beta}\right)\left(\frac{\lambda}{\mu}\right)^3 P_0 + \frac{\lambda}{\lambda+\beta} \cdot \frac{\alpha}{\lambda+\beta} \cdot \left(\frac{\lambda}{\mu}\right)^2 P$$

$$= \left(1+\frac{\alpha}{\lambda+\beta}\right)^2 \left(\frac{\lambda}{\mu}\right)^3 P_0 + \frac{\alpha}{\lambda+\beta} \cdot \frac{\lambda}{\lambda+\beta} \cdot \left(\frac{\lambda}{\mu}\right)^2 P_0$$

(7): $P_{30} = \frac{\lambda}{\beta}\cdot\frac{\alpha}{\lambda+\beta}\cdot\left(1+\frac{\alpha}{\lambda+\beta}\right)\left(\frac{\lambda}{\mu}\right)^2 P_0 + \frac{\lambda}{\beta}\cdot\frac{\lambda}{\lambda+\beta}\cdot\frac{\alpha}{\lambda+\beta}\cdot\frac{\lambda}{\mu} P_0$

$$+ \frac{\alpha}{\beta}\cdot\left(1+\frac{\alpha}{\lambda+\beta}\right)^2\left(\frac{\lambda}{\mu}\right)^3 P_0 + \frac{\alpha}{\beta}\cdot\frac{\alpha}{\lambda+\beta}\cdot\frac{\lambda}{\lambda+\beta}\cdot\left(\frac{\lambda}{\mu}\right)^2 P_0$$

P_0 volgt uit:

2 $P_0 \Big\{ 1 + \frac{\lambda}{\mu}\cdot\left(\frac{\alpha}{\lambda+\beta}+1\right) + \frac{\lambda}{\lambda+\beta}\cdot\frac{\alpha}{\lambda+\beta}\cdot\frac{\lambda}{\mu} + \left(1+\frac{\alpha}{\lambda+\beta}\right)^2\left(\frac{\lambda}{\mu}\right)^2$

$$+ \left(1+\frac{\alpha}{\lambda+\beta}\right)^2\left(\frac{\lambda}{\mu}\right)^3 + \frac{\alpha}{\lambda+\beta}\cdot\frac{\lambda}{\lambda+\beta}\cdot\left(\frac{\lambda}{\mu}\right)^2 + \frac{\lambda}{\beta}\cdot\frac{\alpha}{\lambda+\beta}\left(1+\frac{\alpha}{\lambda+\beta}\right)\left(\frac{\lambda}{\mu}\right)^2$$

$$+ \frac{\lambda}{\beta}\cdot\frac{\lambda}{\lambda+\beta}\cdot\frac{\alpha}{\lambda+\beta}\cdot\frac{\lambda}{\mu} + \frac{\alpha}{\beta}\left(1+\frac{\alpha}{\lambda+\beta}\right)^2\left(\frac{\lambda}{\mu}\right)^3 + \frac{\alpha}{\beta}\cdot\frac{\alpha}{\lambda+\beta}\cdot\frac{\lambda}{\lambda+\beta}\left(\frac{\lambda}{\mu}\right)^2 \Big\} = 1$$

(c) $\mu(P_{11}+P_{21}+P_{31})$

2 $\mu(P_{11}+P_{21}+P_{31}) = \lambda P_0 + \lambda(P_{10}+P_{11}) + \lambda(P_{20}+P_{21})$

(d) $\cancel{\overline{W} = \frac{(P_{20}+P_{21}) + 2(P_{30}+P_{31})}{\mu(P_{11}+P_{21}+P_{31})}}$ (ook goed: $\cancel{\frac{P_{21} + 2(P_{20}+P_{31}) + 3P_{30}}{\mu(P_{11}+P_{21}+P_{21}}}$)

2 $\overline{N} = 2(P_{20}+P_{21}) + 3(P_{30}+P_{31}) + (P_{10}+P_{11})$

2 (e) $P_{11}+P_{21}+P_{31}$

2 (f) $1 - P_0 - P_{11} - P_{21} - P_{31} = P_{10} + P_{20} + P_{30}$

2 (g) $1 - e^{-\lambda t}$