Complex Function Theory (191520252)

Thursday, 23 June 2011, 08.45 - 11.45 am

- An explanation to every answer is required.
- You can make use of calculator.
- Exercise 7 can be skipped in case of homework bonus.

1. (a) Find all $z \in \mathbb{C}$ for which $Re(e^{iz}) = cos(z)$.

   (b) For which $z \in \mathbb{C}$ is $Log(\frac{z-1}{z+1})$ analytic?

2. Describe the stereographic projection on the Riemann sphere of the following sets.

   (a) the disk $\{z \in \mathbb{C} : |z| < \frac{1}{2}\}$.

   (b) the line $\{z \in \mathbb{C} : z = x + ix \text{ with } x \in \mathbb{R}\}$

3. The function $f(z)$ is analytic on $\mathbb{C}$ with $Re(f) = u$ and $Im(f) = v$.

   (a) Denote the Cauchy-Riemann equations for u and v. Express $f'(z)$ in terms of the partial derivatives of v.

   (b) Determine $f(z)$ as a function of z in the case $\frac{\partial v}{\partial x} = 12xy - 6x$ and $f(0) = 3 - 2i$ and $f'(0) = 1$ (hint: try first to determine $f'(z)$).

4. Find the Laurent series for the function

$$\frac{z}{(z+1)(z-2)}$$

in each of following domains

   (a) $\{z \in \mathbb{C} : |z| < 1\}$

   (b) $\{z \in \mathbb{C} : 2 < |z|\}$

5. (a) Find and classify the isolated singularities of

$$f(z) = \frac{e^z}{1 + e^{3z}}$$

   (b) Verify with the aid of residues

$$\int_{-\infty}^{\infty} \frac{e^x}{1 + e^{3x}} \, \mathrm{d}x = \frac{2\pi}{3\sqrt{3}}$$

(hint: apply the residue theorem for a rectangle with height $\frac{2\pi}{3}$ which is based on the real axis)

6. Let m be a positive real number. Prove that the Fourier transform of the function

$$F(t) = \frac{1}{t^2 + m^2}$$

is given by

$$G(\omega) = \frac{1}{2m} e^{-m\omega}$$

for $\omega > 0$.

7. (a) Formulate the 'Argument Principle' for meromorphic functions defined on and inside a simple closed contour.

(b) Prove that the composition of the two Möbius transformations is again a Möbius transformation.

Grading points

1. (a) 3 (b) 3	2. (a) 2 (b) 2	3. (a) 2 (b) 3	4. (a) 2 (b) 2	5. (a) 3 (b) 4	6. 6	7. (a) 2 (b) 2

Total: 36+4=40 points.