UNIVERSITEIT TWENTE.

Complex Function Theory (3EC version EN)

Thursday 26 May 2016, 8.45 - 11.45 uur

Course code 201500405

- An explanation to every answer is required.
- Use of a (graphing) calculator is allowed.

1. a) Solve in $\mathbb{C}$:

$$\mathrm{Log}(z) = 1 + \frac{\pi}{4}i.$$

[Log is the principal value of the logarithm]

b) Draw in one figure:
- the level set at level 1 for $\mathrm{Re}\,(\mathrm{Log}(z))$,
- the level set at level $\pi/4$ for $\mathrm{Im}\,(\mathrm{Log}(z))$.

c) Deduce by using the Cauchy-Riemann Relations that the level sets in b) intersect perpendicular.

2. Suppose $f : \mathbb{C} \to \mathbb{C}$ is entire with $\mathrm{Im}\, f(z) \geq 0$ for all $z \in \mathbb{C}$. Show that f is a constant function.

3. The contour Γ is the unit circle $|z| = 1$, traversed once in the positive sense. Calculate

$$\int_{\Gamma} \frac{\sin z}{z^2(z-4)} \mathrm{d}z.$$

4. Find the Laurent series of

$$\frac{1}{(z+1)(z+2)}$$

for the following regions:

a) $1 < |z| < 2$ (in powers of z),

b) $0 < |z+2| < 1$ (in powers of $z+2$).

5. Evaluate by means of the Cauchy residue theorem

$$\mathrm{p.v.} \int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)^2} \mathrm{d}x.$$

[p. v. is the principle value]

P.T.O.

6. Find the number of roots of the polynomial $p(z) = z^4 + z^3 + 1$ in the disk $|z| < 3/2$.

7. a) Give the inverse formula of the Laplace transform.

[The Laplace transform of $F(t)$ is $\int_0^\infty F(t)e^{-zt}dt$.]

b) The Laplace transform of the function $F(t)$ is given by

$$\frac{1}{z+3}.$$

Calculate $F(t)$ using the inverse formula.

Grading points

1 a: 2 b: 2 c: 2	2: 4	3: 5	4 a: 3 b: 3	5: 7	6: 4	7 a: 1 b: 3

Total: $36 + 4 = 40$ points

End