Complex Function Theory (3EC version EN) Thursday 26 May 2016, 8.45 - 11.45 uur Course code 201500405

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• An explanation to every answer is required.

• Use of a (graphing) calculator is allowed.

1. a) Solve in \mathbb{C} :

$$\operatorname{Log}(z) = 1 + \frac{\pi}{4}i.$$

[Log is the principal value of the logarithm]

- b) Draw in one figure:
 - the level set at level 1 for $\operatorname{Re}(\operatorname{Log}(z))$,
 - the level set at level $\pi/4$ for Im (Log(z)).
- c) Deduce by using the Cauchy-Riemann Relations that the level sets in b) intersect perpendicular.
- 2. Suppose $f : \mathbb{C} \to \mathbb{C}$ is entire with $\operatorname{Im} f(z) \ge 0$ for all $z \in \mathbb{C}$. Show that f is a constant function.
- 3. The contour Γ is the unit circle |z| = 1, traversed once in the positive sense. Calculate

$$\int\limits_{\Gamma} \frac{\sin z}{z^2(z-4)} \mathrm{d}z$$

4. Find the Laurent series of

$$\frac{1}{(z+1)(z+2)}$$

for the following regions:

a) 1 < |z| < 2 (in powers of z),

- b) 0 < |z+2| < 1 (in powers of z+2).
- 5. Evaluate by means of the Cauchy residue theorem

p.v.
$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)^2} \mathrm{d}x.$$

[p. v. is the principle value]



- 6. Find the number of roots of the polynomial $p(z) = z^4 + z^3 + 1$ in the disk |z| < 3/2.
- 7. a) Give the inverse formula of the Laplace transform. [The Laplace transform of F(t) is $\int_{0}^{\infty} F(t)e^{-zt}dt$.]
 - b) The Laplace transform of the function F(t) is given by

$$\frac{1}{z+3}$$
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Calculate F(t) using the inverse formula.

Grading points

1 a: 2	2:4	3: 5	4 a: 3	5:7	6:4	7 a: 1	
b: 2			b: 3		1.1	b: 3	Sec.
c: 2							

Total: 36 + 4 = 40 points

End