UNIVERSITEIT TWENTE.

Complex Function Theory

Thursday 1 June 2017, 8.45 - 11.45 a.m.

Course code 201500405

- An explanation to every answer is required.
- Use of a (graphing) calculator is allowed.

1. Solve for $z \in \mathbb{C} \setminus \{0\}$:

   a) $\sinh(z) \in \mathbb{R}$,

   b) $z^i = 1$.

2. Suppose that $f(z) = f(x+iy) = u(x,y) + iv(x,y)$ is analytic in $\mathbb{C}$.

   a) Formulate the Cauchy-Riemann equations for $f(z)$.

   b) Suppose that $g(z) = v(x,y) + iu(x,y)$ is also analytic in $\mathbb{C}$. Show that in this case both $u(x,y)$ and $v(x,y)$ are constant.

3. The contour Γ is the unit circle $|z| = 1$, traversed once in the positive sense. Calculate:

   a) $\displaystyle\int_\Gamma \frac{1}{\bar{z}}\mathrm{d}z$,

   b) $\displaystyle\int_\Gamma \frac{\cos(z)}{z(z-3)}\mathrm{d}z$,

   c) $\displaystyle\int_\Gamma e^{1/z}\mathrm{d}z$.

P.T.O.

4. Consider

$$f(z) = \frac{1}{z(z-1)(z+3)}.$$

This function is expanded in three different Laurent series involving powers of z.

a) One of the three domains is the annulus $\{z \in \mathbb{C} \mid 0 < |z| < 1\}$. Give the Laurent series for this domain.

b) One of the Laurent series of $f(z)$ is given by:

$$\sum_{n=-\infty}^{-3} c_n z^n.$$

To which domain belongs this series?

5. Evaluate by means of the Cauchy residue theorem:

$$\text{p.v.} \int_0^\infty \frac{dx}{(x^2+a^2)^2}, \qquad \text{with } a > 0.$$

[p. v. is the principle value]

6. Prove that all zeros of $z^4 + z^3 + 1$ are lying in the annulus $\{z \in \mathbb{C} \mid 3/4 < |z| < 3/2\}$.

7. a) Formulate the inverse formula of the Laplace transform

$$\mathcal{L}\{F\}(s) = \int_{t=0}^{\infty} F(t) e^{-st} \mathrm{d}t$$

of the function $F(t)$.

b) Use this formula to find $F(t)$ in case the Laplace transform of $F(t)$ is given by the function

$$\frac{s}{s^2+4}.$$

Grading points

1 a: 2 b: 3	2 a: 1 b: 3	3 a: 2 1 b: 2 c: 2	4 a: 4 2 b: 2	5: 6	6: 4	7 a: 1 b: 4 2

Total: $36 + 4 = 40$ points

3 2 3

3 4 4 2

End