# UNIVERSITEIT TWENTE.

**Complex Function Theory**
Monday 24 July 2017, 1.45 - 4.45 p.m.
Course code 201500405

- An explanation to every answer is required.
- Use of a (graphing) calculator is allowed.

1. Solve for $z \in \mathbb{C} \setminus \{0\}$:

   a) $e^z = e^{iz}$,

   b) $\operatorname{Log}^2 z - 2\operatorname{Log} z + 2 = 0$.
   [$\operatorname{Log} z$ is the principle value of the logarithm function]

2. Consider the harmonic function $u(x, y) = \sin(x)\cosh(y)$.

   a) Find all harmonic conjugates of $u(x, y)$ defined on $\mathbb{R}^2$.

   b) Let $z = x + iy$. Determine the analytic function $f(z)$ with $\operatorname{Re}(f(z)) = u(x, y)$ and $f(0) = i$. Express $f(z)$ in terms of $z$.

3. Calculate the following contour integrals. The contour $C$ is traversed once in the counter-clockwise direction.

   a) $\displaystyle\int_C \frac{\cos(z)}{(z-1)^3(z-5)^2}\,dz$, with $C$ the circle $|z-4| = 2$.

   b) $\displaystyle\int_C \frac{e^{5z}}{z^3}\,dz$, with $C$ the circle $|z| = 1$.

P.T.O.

4. Consider

$$f(z) = \frac{1}{z^2 + z}.$$

Find the Laurent series expansion in the regions

a) $0 < |z| < 1$,

b) $1 < |z|$,

c) $0 < |z+1| < 1$,

d) $1 < |z+1|$.

5. Evaluate by means of the Cauchy residue theorem:

$$\text{p. v.} \int_{-\infty}^{\infty} \frac{x^2+1}{x^4+4} dx.$$

[p. v. is the principle value]

6. a) Formulate the Argument Principle.

b) Evaluate

$$\int_C \frac{4z^3}{z^4+16} dz,$$

with $C = \{z \in \mathbb{C} \,|\, |z| = 3\}$ positively oriented.

c) Prove that the equation $z^3 + 9z + 27 = 0$ has no roots in the disk $|z| < 2$.

7. a) Formulate the inverse formula for the Laplace transform

$$\mathcal{L}\{F\}(s) = \int_{t=0}^{\infty} F(t)e^{-st} dt$$

of the function $F(t)$.

b) Use this formula to find $F(t)$ in case the Laplace transform of $F(t)$ is given by the function

$$\frac{1}{(s+1)^2}.$$

**Grading points**

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|---|---|---|---|---|---|---|
| 1 a: 2<br>b: 3 | 2 a: 1<br>b: 3 | 3 a: 3<br>b: 3 | 4 a: 1<br>b: 1<br>c: 1<br>d: 1 | 5: 6 | 6 a: 2<br>b: 2<br>c: 2 | 7 a: 1<br>b: 4 |

Total: 36 + 4 = 40 points

End