UNIVERSITEIT TWENTE.

Complex Function Theory

Thursday 31 May 2018, 8.45 - 11.45 a.m.

Course code 201500405

- An explanation to every answer is required.
- Use of a (graphing) calculator is allowed.

1. a) Find the (largest) domain where $\operatorname{Log}\left(\frac{z-1}{z}\right)$ is analytic.

   b) Calculate $\frac{\mathrm{d}}{\mathrm{d}z}\operatorname{Log}\left(\frac{z-1}{z}\right)$ in that domain.

2. a) Show that $u(x,y) = x^3 - 3xy^2 + 2y$ is harmonic in $\mathbb{R}^2$.

   b) Suppose $z = x + iy$ and $f(z)$ is an entire function with $\operatorname{Re}(f(z)) = u(x,y)$ and $f(1+i) = i$. Find $f(z)$ as a function expressed in z.

3. Let $f(z)$ be entire (analytic in the whole plane) and suppose that $\operatorname{Re}(f(z)) \leq M$ for all z, where M is a fixed positive number. Prove that f is a constant function.

4. Given the function $f(z) = \dfrac{z}{z^2 - 9}$.

   a) Expand $f(z)$ in a Taylor series around $z = 0$ (MacLaurin series). In what disk centered at $z = 0$ is the series convergent?

   b) Expand $f(z)$ in a Laurent series in powers of $z - 3$ valued in an annular domain containing the point $z = 0$.

5. Using the technique of residues, compute

$$\text{p. v.} \int_{-\infty}^{\infty} \frac{\cos(3x)}{x-1}\mathrm{d}x.$$

   [p. v. is the principle value]

P.T.O.

6. Given the equation $20z^2 + 30 - e^z = 0$.

   a) Show that if z solves the equation, so does $\bar{z}$.

   b) Prove that the equation has two solutions inside $|z| = 2$.

7. a) Given the smooth function $F(t)$ for $t \geq 0$. Give the formula for the Laplace transform $\mathcal{L}\{F(t)\}$ as well as the formula for the inverse transformation.

   b) Use the Laplace transform to solve the initial value problem:

$$\begin{cases} y'' + 2y' + y = 2, \\ y(0) = 1 \text{ and } y'(0) = 1. \end{cases}$$

Grading points

1 a: 3 b: 1	2 a: 1 b: 4	3: 4	4 a: 3 b: 3	5: 7	6 a: 1 b: 5	7 a: 1 b: 3

Total: $36 + 4 = 40$ points

End