Re-Exam Complex Function Theory
Code 2018-201500405

Date	:	Thursday, October 10, 2019
Place	:	CR-3D
Time	:	08.45 – 10.45

All answers must be motivated.
The use of a pocket calculator is not allowed.

1. On the non-empty domain Ω the analytic function f is given.
   - 3 (a) Prove that $g(z) := \sin(f(z))$ is analytic on Ω.
   - 3 (b) Give necessary and sufficient conditions in terms of f such that g is bounded on Ω.
2. Given is the function
$$u(x, y) = -2xy - 5y + i. \tag{1}$$
   - 2 (a) Show that u is harmonic on $\mathbb{R}^2$.
   - 3 (b) Suppose that $z = x + iy$ and $f(z) = u(x, y) + iv(x, y)$, with f entire and u given by (1). Find $f(z)$, as function of z, when additionally it is given that $f(0) = 1$.

4 3. For which $z \in \mathbb{C}$ do we have that $\mathrm{Log}(e^z) = e^{\mathrm{Log}(z)}$?

3 4. Determine the integral of $\frac{\cos(z)}{z-1}$ along the closed curve $\gamma = \{\frac{e^{2it}}{e^{it}-2} \mid t \in [0, 2\pi]\}$, see Figure 1 for its graph and orientation.

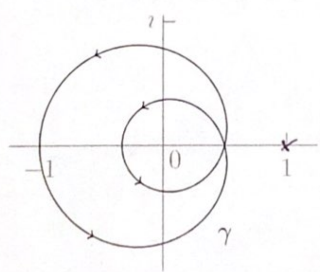

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Figure 1: The curve γ.

P.T.O.

$\frac{f^{(j)}(z_0)}{j!}(z-z_0)^j$

$\sum_{j=0}^{\infty}$

5. 2 (a) Determine the Taylor series of $\cos(z)$ around $z = 0$.

   3 (b) Prove that the function

$$h(z) = \cos(\sqrt{z})$$

   is entire. Here $\sqrt{z}$ is obtained via the principle value.

3 6. Let $f(z)$ be given as

$$f(z) = \frac{z^2 \sin(z)}{(z+1)(z+3)}.$$

Principal argument

Determine the integral

$$\int_{C_2} \frac{f'(z)}{f(z)} dz,$$

$\frac{1}{2\pi i}\int_{C_2} \frac{f'(z)}{f(z)} dz = N_0(f) - N_p(f)$.

where C_2 is the circle, positively oriented, with centre the origin and radius 2.

7. Consider the function $f(z) = z^5 - e^{z-2}$.

   3 (a) How many zeros has f within the unit circle? Rouché. 5?

   3 (b) Define for $\alpha, \beta \in \mathbb{R}$ with $\alpha < \beta$ the strip $S_{\alpha,\beta}$ as

$$S_{\alpha,\beta} := \{z \in \mathbb{C} \mid \mathrm{Re}(z) \in [\alpha, \beta]\}.$$

   Show that there does not exists a strip $S_{\alpha,\beta}$ such that f has infinitely many zeros within $S_{\alpha,\beta}$.

$e^{iz} = \cos z + i \sin z$.

4 8. Determine the following integral

$$\text{p.v.} \int_{-\infty}^{\infty} \frac{\sin(x)}{x} dx.$$

$\frac{e^{iz}}{2iz} - \frac{e^{-iz}}{2iz}$

Points[1]

Ex. 1		Ex. 2		Ex. 3	Ex. 4	Ex. 5		Ex. 6	Ex. 7		Ex. 8
a	3	a	2	4	3	a	2	3	a	3	4
b	3	b	3			b	3		b	3	
	3		5					3		3	4

[1]Total: $36 + 4 = 40$ points