Exam Complex Function theory
Code 2019-201500380

Date	:	Thursday 28 May 2020
Place	:	Home
Time	:	13.45 – 15.45

All answers must be motivated.
If you have questions, then you can use scat function of Canvas or phone: 053-4893464
The use of the book is allowed. However, it is not allowed to let you be assisted or to help any other, see also the statement at the end of the exam.

1. Determine all values of $(-1)^i$.

2. Let $D \subset \mathbb{C}$ be a given domain let $f : D \mapsto \mathbb{C}$ be analytic on D. Show that if $\overline{f(z)}$ is analytic on D as well, then f is constant.

3. Determine the outcome of the following integral
$$\int_\Gamma \frac{z^4 + 5z^3}{(z-1)^2} dz,$$
where Γ is the contour $\Gamma = \{1 + e^{i\theta} \mid \theta \in [0, 2\pi]\}$.

4. The function $q : \mathbb{R} \mapsto \mathbb{R}$ given by $q(x) = x^2$ has the property that its values never become negative. In this exercise, we begin to show that this cannot hold for an entire function.

   Let $f : \mathbb{C} \mapsto \mathbb{C}$ be an entire function with a double zero at $z = 0$. Show that there exists a point $w \in \mathbb{C}$ such that the real part of $f(w)$ is negative, i.e., $\mathrm{Re}(f(w)) \in (-\infty, 0)$.

5. Determine whether the following assertions are true or false:

   (a) The number of solutions of the equation
   $$\frac{z^5}{z^6 + 1} = 1$$
   is equal to 6.

   (b) If f and g are entire functions, then $h(z)$ defined as
   $$h(z) = f(z)^{g(z)}, \qquad z \in \mathbb{C}$$
   is also entire.

P.T.O.

6. The function

$$f(z) = \frac{z^3(z+3)}{ze^{-2z} + z + 1}$$

is drawn for z on the unit circle (running in the positive direction) in Figure 1.

(a) How many poles and zeros (counted with multiplicity) does f have inside the unit circle?

(b) If the arrows in Figure 1 would point in the opposite direction, would that change your answer in item a?

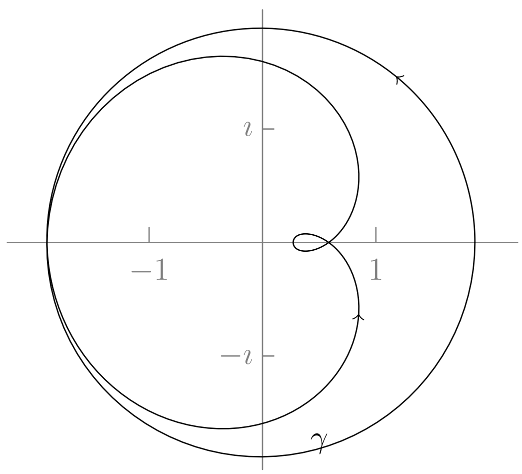

Figure 1: The curve γ.

7. Determine the integral

$$\text{P.V.} \int_{-\infty}^{\infty} \frac{x^2+4}{(x^2+1)(x^2+9)} dx,$$

where P.V. stands for principle value.

8. Consider the function

$$h(z) = z^4 + 1 + 0.8e^{-z}. \tag{1}$$

Let Ω_R^+ be the half disc $\Omega_R^+ = \{z \in \mathbb{C} \mid \text{Re}(z) \geq 0, |z| \leq R\}$, with $R > 0$.

(a) How many zeros does h have in Ω_2^+?

(b) How many zeros does h have with positive real part?

Points[1]

Ex. 1	Ex. 2	Ex. 3	Ex. 4	Ex. 5		Ex. 6		Ex. 7	Ex. 8	
3	4	4	4	a	3	a	3	5	a	3
				b	3	b	2		b	2

[1] Total: $36 + 4 = 40$ points

Statement:

Copy the statement below on a separate sheet of paper, and hand it in together with your exam solutions.

I, `insert your name`, hereby declare that I have made this test to the best of my own ability. I personally guarantee that I have made this test without seeking or accepting the help of any source not explicitly allowed by the conditions of the test.

Date and place: . . . .