Graph Theory (191520751)
april 16, 2015, 8.45 – 11.45 h

Motivate your answers.
All graphs are simple.

1. True or false? (Provide arguments! 2 points per item)

   a) If G is a simple disconnected graph, its complement G^c is connected.

   b) A simple graph with n nodes and n edges contains exactly one cycle.

   c) A simple connected graph with n nodes and n edges contains exactly one cycle.

   d) A simple connected graph with n nodes and $n+1$ edges contains exactly two cycles.

2. Let $T = (V, E)$ be a tree. Show that the following procedure computes a longest path P in T:
   Start with a node $u \in V$. Compute a node v at maximum distance from u. Then compute a node w at maximum distance from v. Let P be the path from v to w.

3. Sketch a proof of $\tau(K_n) = n^{n-2}$. (τ = number of spanning trees.)

4. Show that the d-dimensional cube graph Q_d is hamiltonian.

5. State Tutte's Theorem on perfect matchings.
   Derive a min-max formula for the size of a maximum matching in a graph G.
   (No proof required, but you can earn 3 extra points for providing one.)

6. G is a simple 3-regular hamiltonian graph. Show that $\chi'(G) = 3$.

7. Assume that $\chi(G) = k$. Show that G contains at least k nodes with degree $\geq k - 1$.

Points: 36+4 = 40

1: 8	2: 5	3: 5	4: 4	5: 5	6: 4	7: 5