Exam Graph Theory (191520751)
Tuesday jan 26, 2016, 8.45 – 11.45 uur

All graphs are simple.
Motivate your answers.

1. Show that a graph G is bipartite if and only if every subgraph H of G contains an independent set of size $\nu(H)/2$.
(An *independent set* is a set of pairwise nonadjacent nodes.)

2. Let G be a connected graph with at least 3 nodes. Show that there are two (different) nodes x and y at distance $d(x, y) \leq 2$ such that $G - \{x, y\}$ is still connected.
(Hint: Consider a spanning tree.)

3. Show that the k-dim cube graph Q_k is k-connected.

4. G is a simple 3-regular hamiltonian graph. Show that $\chi'(G) = 3$.

5. Let $G = (V, E)$ be a graph on n vertices.
a) Show: If $M \subseteq E$ is a matching and $S \subseteq V$, then

$$|M| \leq \frac{1}{2}(n - o(G - S) + |S|).$$

(Reminder: $o(G - S)$ is the number of odd components in $G \backslash S$.)

b) In case G is bipartite and $S \subseteq V$ is a vertex cover, what does the above inequality say?

6. Show that a k-critical graph contains at least k nodes of degree $\geq k-1$.

7. G is a simple graph on n nodes. Show: $\chi(G) \cdot \chi(G^c) \geq n$.
(G^c is the complement of G.)

Points (36+4=40):

1: 5	2: 5	3: 5	4: 5	5: 6	6: 5	7: 5