Exam Graph Theory (191520751)
Friday april 15, 2016, 13.45 – 16.45 hrs

Motivate your answers.

1. Let G be a connected graph with at least 3 nodes. Show that there are two distinct nodes i, j at distance $d_{ij} \leq 2$ such that $G \backslash \{i, j\}$ is (still) connected. [Hint: Consider a spanning tree of G.]

2. Assume that G is a simple 5-regular graph with $\kappa(G) = 2$. Show: $\kappa'(G) \leq 4$. [Reminder: κ, κ' are node and edge connectivity, resp.]

3. Let $G = (V, E)$ be a simple graph, $M \subseteq E$ a maximum matching and $K \subseteq V$ a minimum node cover. Show that $\cancel{|K| \leq |M| \leq 2|K|}$. $|M| \leq |K| \leq 2|M|$
Provide examples for both extreme cases ($|K| = |M|$ and $|K| = 2|M|$).

4. Which labeled tree corresponds to the Prüfer code (4,3,5,3,4,5)? (Reminder: The Prüfer code identifies labeled trees on n nodes with sequences in $\{1, \ldots, n\}^{n-2}$.)

5. The number of spanning trees of K_n is known to be $\tau(K_n) = n^{n-2}$. Conclude that $\tau(K_n \backslash e) = (n-2)n^{n-3}$.

6. Assume G is a simple graph with $\chi(G) = k$ and consider a fixed proper k-coloring of G. Show that for each color i there is some vertex of color i which is adjacent to all other $k-1$ colors.

7. Give a proof of Euler's formula " $\nu - \epsilon + \phi = 2$ " for planar graphs.

Normering (36+4=40):

1: 6	2: 5	3: 5	4: 5	5: 5	6: 5	7: 5