Kenmerk : AM2018/DMMP/003/ha

Course	:	**Graph Theory**
Date	:	20 April 2018
Time	:	08.45-11.45 uur

Motivate all your answers. The use of electronic devices is not allowed.

In this exam a graph G is a *simple* graph, i.e, G has no loops and two distinct vertices are connected by at most one edge.

For any $x \in \mathbb{R}$, the *floor* of x is denoted by $\lfloor x \rfloor$.

1. [6 pt]
   Let v be a cut vertex of G.
   Prove that v is not a cut vertex in G^c (G^c is the complement of G).

2. [5 pt]
   T is a tree with $\nu \geq 2$ vertices satisfying $\Delta(T) = 3$.
   Let z denote the number of vertices of T with degree 3. Prove that: $z \leq \left\lfloor \frac{\nu - 2}{2} \right\rfloor$.

3. [4 pt]
   Prove that for each graph, $\kappa' \leq \left\lfloor \frac{2\epsilon}{\nu} \right\rfloor$.

4. [4 pt]
   Let G be a bipartite graph with bipartition (X, Y).
   Suppose there exists a $k > 0$ such that for all $x \in X$ and all $y \in Y$: $d(y) \leq k \leq d(x)$.
   Prove that G contains a matching that saturates each $x \in X$.

5. [5 pt]
   Prove that a 3-regular Hamiltonian graph G satisfies $\chi'(G) = 3$.

6. [6 pt]
   Let G be a graph with complement G^c. Show that $\chi(G) + \chi(G^c) \geq 2\sqrt{\nu}$.

7. [6 pt]
   Let G be a 5-regular planar graph with ν vertices, ϵ edges and ϕ faces, where each face is bounded by exactly three edges. Determine ν, ϵ and ϕ.

Total: 36 + 4 = 40 points