Exam Graph Theory (191520751)

Monday, april 15, 2019, 13.45 – 16.45 hrs

Motivate your answers.

1. Let $G = (V, E)$ be a graph with minimum degree $\delta \geq k$. Show that G must contain a path of length k.

2. Let $G = (V, E)$ be a triangle-free graph on $2k$ vertices. Show that G has at most k^2 edges.

3. Let G be a graph. Show that either G or its complement G^c is connected. Does this also hold for 2-connectedness?

4. Use Tutte's Theorem to prove the following result: $o(G-S) \leq |S|$
   Theorem (König): A bipartite graph $G = (V, E)$ has a perfect matching if and only if there is no vertex cover $S \subseteq V$ of size less than $|V|/2$.

5. Show that $\chi' = \Delta$ for regular bipartite graphs. You may use König's Theorem from Exercise 4.

6. Let $G = (V, E)$ be critical (i.e., k-critical for some k).
   Show: If $S = \{u, v\}$ is a vertex cut then $uv \notin E$.
   [Provide a direct proof without using any results from the book.]

7. Let G be planar with minimum degree $\delta \geq 3$ and at most 11 faces. Conclude that G must have a face of degree at most 4.

Points (36+4=40):

1: 5	2: 5	3: 5	4: 6	5: 5	6: 5	7: 5