Exam Graph Theory (191520751)

University of Twente
April 16, 2021, 13:45–16:45

This exam has 7 exercises.
Motivate/justify all your answers by explaining how you got to them.
You may not use any electronic device or lecture materials, books, notes, et cetera.
Don't forget to turn off your cell phone!

1. Let G be a simple 2-connected graph. We add one new vertex to G and join it to exactly one vertex of G by an edge. Let G' denote the obtained graph.
   (a) Prove that G' has exactly one cut vertex.
   (b) Prove the following statement: If a simple graph H with $\kappa(H) = 1$ can be contracted to a 2-connected graph by contracting just one edge, then $\delta(H) = 1$.

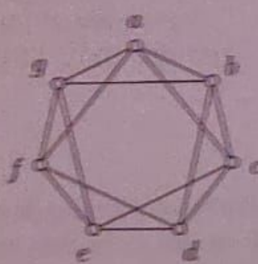
2. Let G be a simple 3-connected graph with 10 vertices and 19 edges. Determine $\delta(G)$.

3. Let G be a simple bipartite graph with bipartition (X, Y) where both X and Y are nonempty. Suppose that $\delta(G) > 0$ and that each vertex of Y has degree $\delta(G)$. Prove that G has a matching saturating all vertices of X.

4. For each of the following polynomials, show that it is **not** the chromatic polynomial of a simple graph.
   (a) $k^5 - 5k^4 + 10k^3 - 3k^2 - 3k$
   (b) $k^4 - 3k^3 + k^2$
   (c) $k(k-1)^{20}(k-3)^2$

Please turn over.

5. Let G be the following graph:



(a) Determine $\chi(G)$.

(b) Prove that $\chi(G-v) < \chi(G)$ for every vertex v of G.

(c) Determine whether G is a critical graph or not.

6. Let G be a simple 3-regular hamiltonian graph.

(a) Prove that G is 3-edge-colourable.

(b) Prove that for each edge e of G there is a perfect matching in G containing e.

7. Let G be a path on $\nu \geq 1$ vertices and let G^c denote the complement of G. For which values of ν are both G and G^c planar graphs?

Norm:

	1		2	3	4			5			6		7
Exercise	a	b			a	b	c	a	b	c	a	b	
Points	2	3	4	5	2	2	2	2	2	3	3	2	4

Total: 36 points. Your grade is $\frac{1}{4}\times$ (your total score of points plus 4) (rounded).