Graph Theory (152075)

wednesday january 28, 2009, 13.30 – 16.30 hrs

Motivate your answers.

1. Do there exist simple graphs with degree sequence
   (a) (7,7,7,7,5,5,5,5)
   (b) (7,5,5,4,3,2,1,1)?

2. Let G be a simple graph and let G^c denote the complementary graph. Show: $diam(G) \geq 3 \Rightarrow diam(G^c) \leq 3$. (Here, $diam$ = diameter.)

3. Explain how Cayleys formula $\tau(K_n) = n^{n-2}$ is derived.

4. Show: If G is hamiltonian and $\emptyset \neq S \subset V$, then $\omega(G - S) \leq |S|$.

5. Let $G = (V, E)$ be a graph, $M \subseteq E$ be a matching and $S \subset V$. Show:
$$|M| \leq \frac{1}{2}[\nu - (o(G - S) - |S|)].$$

6. Determine the chromatic polynomial of C_5 (the cycle of length 5). Compute the probability that a random 5-coloring of C_5 is proper.

7. Show: If $(S, \overline{S})$ and $(T, \overline{T})$ are minimum cuts in a network, then so are $(S \cup T, \overline{S \cup T})$ and $(S \cap T, \overline{S \cap T})$.

Points:

1: 5	2: 5	3: 6	4.: 5	5: 5	6: 5	7: 5

Total: $36 + 4 = 40$ points.