Random Signals and Systems (157108)

Hajek's lecturenotes and additional course notes and your own notes & homework can be consulted (but other notes are not allowed)

Date: 14-04-2009
Place: Vrijhof, VR6
Time: 9:00–12:00

1. Suppose X, Y are random variables with $X, Y \in \{1, 2, 3\}$ and that its joint pmf $p(x, y)$ is

$$\begin{array}{lll} p(1,1) = 1/9, & p(2,1) = 1/3, & p(3,1) = 1/9, \\ p(1,2) = 1/9, & p(2,2) = 0, & p(3,2) = 1/(18), \\ p(1,3) = 0, & p(2,3) = 1/6, & p(3,3) = 1/9. \end{array}$$

   Are X and Y independent?

2. Let $\{Z_n\}_{n\in\mathbb{N}}$ be an iid sequence with $\mathrm{E}(Z_n) = 0$ and $\mathrm{E}(Z_n^2) = 1$. Let $X_0 = 0$ and $X_{n+1} := X_n + Z_n$ for $n > 0$. Is the process $\{X_n\}_{n\in\mathbb{N}}$ a martingale?

3. Let $Y = \int_0^2 X_t \, dt$ and assume $\{X_t\}_{t\in\mathbb{R}}$ is zero mean WSS with $R_X(\tau) = 1 - |\tau|$ for $\tau \in [-1, 1]$ and zero elsewhere.
   (a) Which proposition(s) in Hajek gaurantee that $Y = \int_0^2 X_t \, dt$ is well defined (in some sense)?
   (b) Determine $\mathrm{var}(Y)$.

4. Let X and Y be independent exponential distributed random variables both with parameter λ. Let $U = X + Y$, $V = X - Y$. Determine the joint pdf $F_{UV}(u, v)$.

5. Consider

$$f_{XY}(x, y) = \begin{cases} \dfrac{e^{-x/y}\, e^{-y}}{y} & \text{if } 0 < x < \infty, 0 < y < \infty \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Determine $f_{X|Y}(x|y)$.
   (b) Determine $\mathrm{E}(X|Y)$.

6. Consider the discrete process $\{X_n\}_{n\in\mathbb{N}}$ with $X_0 = 0$ and

$$X_n = \frac{1}{2} X_{n-1} + W_n, \quad n > 0,$$

   where $\{W_n\}_{n\in\mathbb{N}}$ is a white noise process. Does $\lim_{n\to\infty} X_n$ exist in m.s. sense? (Referring to examples in Hajek is not allowed. You have to prove it from scratch.)

7. Suppose $\{X_t\}_{t\in\mathbb{R}}$ is zero mean WSS with correlation function R_X. Let $\alpha > 0$. Show that $P(|X(t+\tau) - X(t)| \geq \alpha) \leq 2(R_X(0) - R(\tau))/\alpha^2$.

8. Suppose $\{X_n, Y_n\}_{n\in\mathbb{Z}}$ are zero mean jointly WSS stochastic processes with well defined spectral densities $S_X(\omega)$, $S_Y(\omega)$. Are X_n and Y_k uncorrelated processes (meaning $C_{XY}(\tau) = 0 \; \forall \tau \in \mathbb{Z}$) if and only if $S_{X+Y}(\omega) = S_X(\omega) + S_Y(\omega)$ for all $\omega \in \mathbb{R}$?

problem:	1	2	3(a)	3(b)	4	5(a)	5(b)	6	7	8
points:	3	3	3	4	5	4	4	4	4	4

Exam grade is $1 + 9p/38$. (Final grade may depend on homework.)