Random Signals and Systems (157108)

The lecture notes are allowed
(other sources are not allowed)

Date: 13-04-2010
Place: Vrijhof, VR6
Time: 13:45–16:45

1. For which $\alpha \in \mathbb{R}$ is

$$R_\alpha = \begin{bmatrix} 1 & 2 \\ 2 & \alpha \end{bmatrix}$$

a possible covariance matrix, and for all these cases define random variables (X_1, X_2) whose covariance matrix is R_α.

2. Let X and Y be iid with probability density $f_X(z) = f_Y(z) = \mathrm{e}^{-z}\,\mathbb{1}(z)$. Let $U = Y/X$ and $V = X$. Determine the joint probability density function $f_{UV}(u, v)$.

3. Let

$$f_{XY}(x, y) = \begin{cases} 1 & 0 < |y| < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Determine $f_Y(y)$
   (b) Determine $f_{X|Y}(x|y)$
   (c) Determine $\mathrm{E}(X|Y)$

4. Suppose $X, Y_1, \ldots, Y_n$ all have zero mean and finite second moment. Let (as in the lecture notes) $\mathrm{E}_{\mathrm{lin}}(X|Z)$ denote the MMSE estimator of X that is linear in Z. Suppose that for every X we have

$$\mathrm{E}_{\mathrm{lin}}(X|Y_1, \ldots, Y_n) = \sum_{k=1}^{n} \mathrm{E}_{\mathrm{lin}}(X|Y_k).$$

Show that $Y_1, \ldots, Y_n$ are then uncorrelated random variables.

5. Let $Y = \int_0^1 X_t\, dt$ and assume that $\{X_t\}_{t\in\mathbb{R}}$ is zero mean WSS with covariance function $R_X(\tau) = 1 - |\tau|$ for $\tau \in [-1, 1]$.

   (a) Compute var(Y)
   (b) Is Y Gaussian if $\{X_t\}_{t\in\mathbb{R}}$ is a Gaussian process?

6. Suppose a zero mean discrete random process $\{U_n\}_{n\in\mathbb{Z}}$ with spectral density

$$S_U(\omega) = \frac{1}{|1 + \frac{1}{2}\mathrm{e}^{-\mathrm{i}\omega}|^2}.$$

Determine the covariance function R_Y of $\{Y_n\}_{n\in\mathbb{Z}}$ defined as

$$Y_n := U_n + \frac{1}{2}U_{n-1}, \qquad n \in \mathbb{Z}.$$

7. Suppose $f_{X_n}(x)$ is the probability density function of X_n and that $f_{X_n}(x) = 0$ for all $|x| \geq 1$ for all n. If $X_n \xrightarrow[n\to\infty]{} 0$ in probability, does $X_n \xrightarrow[n\to\infty]{} 0$ also in mean square?

problem:	1	2	3	4	5	6	7
points:	3	6	3+3+2	4	3+2	3	3

Exam grade is $1 + 9p/p_{\max}$. (Final grade may depend on homework.)