Exam Continuous Optimization

24 January 2022, 13.30–16.30

This closed-book exam consists of 5 questions. Please start each question on a new page, write legibly, and hand in your work with the solutions in the correct order. Good luck!

1. (5 points) Let
$$f(x_1, x_2) = x_1^2 - x_2^2.$$
Compute the Newton direction Δx_{nt} of f at $(1, 2)$ and show this is not a descent direction.

2. (10 points) Consider the equality constrained least squares problem
$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2^2 \\ \text{subject to} & Cx = d, \end{array}$$
Explain how (and why) we can use the KKT optimality conditions to solve this problem by solving a single linear system.

3. (10 points) Derive the Lagrangian, Lagrange dual function, and the Lagrange dual problem of the following optimization problem in x and y:

Check?
$$\begin{array}{ll} \text{minimize} & -\sum_{i=1}^{m} \log(y_i) \\ \text{subject to} & y = b - Ax \end{array}$$

4. (a) (7 points) Consider the function
$$F(x, y) = x^3 + \frac{1}{xy},$$
where we view addition, multiplication, taking the reciprocal, and taking the third power as elementary functions. Show how $\nabla F(1, 2)$ is computed using reverse-mode automatic differentiation by drawing the appropriate diagrams.

   (b) (3 points) Give the cost function for training a neural network using supervised learning, and explain why reverse-mode automatic differentiation is used here.

5. Consider the barrier method for an optimization problem of the form

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leq 0, \quad i = 1, \ldots, m, \end{array}$$

where the functions $f_0, \ldots, f_m$ are convex and twice continuously differentiable. Assume the problem has an optimal solution x^* with objective p^*. Assume furthermore the problem is strictly feasible.

~ (a) (7 points) Show that if $x^*(t)$ is optimal for the centering problem with parameter t, then

$$f_0(x^*(t)) - p^* \leq \frac{m}{t}.$$

V (b) (3 points) Suppose $m = 1000$ and we apply the barrier method with parameters $\mu = 2$, $\epsilon = 1$, and with 1 as the initial value for t. After approximately how many outer iterations does the barrier method terminate?

End of test